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Article

An Application of Manifold-Constrained Hyper-Connection in A Progressive Web App for Sovereign Debt Sustainability Analysis

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Abstract: Importance: The rise in Trinidad and Tobago's (T&T's) public debt from 61.9% of GDP in 2019 to 75.6% by 2024 highlights a fiscal vulnerability facing the country. Like many other Caribbean small open economies, it is highly susceptible to external shocks, that can rapidly escalate debt-to-GDP ratios, threatening long-term economic stability. **Research Gap:** While the IMF provides debt sustainability frameworks, their implementation requires advances mathematical skills and extensive data often unavailable in developing countries. Existing literature identifies vulnerability under current policies but offers limited actionable guidance on the specific fiscal adjustment required to achieve sustainability targets, creating an operational gap between diagnosis and practical planning. **Objective:** This study proposes and designs a computational framework for a PWA to derive the fiscal surplus required to bring the debt-to-GDP ratio to a sustainable level of 60% of GDP in 10 years. **Methodology:** The methodology forecasts 10-year GDP using Manifold-Constrained Hyper-Connections (mHC), computes target debt at 60% of GDP, calculates the difference from current debt, and amortizes this excess debt to determine required annual fiscal surplus. **Key findings:** Achieving the 60% target by 2034 requires reducing debt by US$2,072.57 million, necessitating a steady annual fiscal surplus of US$268.41 million at a 5% discount rate. **Implications:** This study contributes the first empirical mHC application for macroeconomic forecasting, a pragmatic debt sustainability framework operationalized with minimal data, and an accessible tool that integrates technical fiscal analysis for resource-constrained policymakers in developing economies.

Keywords: Progressive Web Apps; Debt Sustainability; Manifold-Constrained Hyper-Connection; Econometrics; Trinidad and Tobago.

1. Introduction

The sustainability of sovereign debt remains a critical issue for economic stability, particularly for small open economies in the Caribbean [1]. These countries are inherently vulnerable to external shocks; including global financial crises, commodity price volatility; that can rapidly escalate public debt-to-GDP ratios to dangerous levels. The ability to conduct rigorous debt sustainability analysis is there essential for informed fiscal policy formulation and maintaining credibility with international financial institutions and markets.

Trinidad and Tobago (T&T) exemplifies this vulnerability. The country's public debt has risen substantially from 61.9% of GDP in 2019 to 75.6% by the end of 2024, reflecting a compounding fiscal problem. While the fiscal stimulus implemented during the COVID-19 pandemic from 2020 to 2023 was a justifiable countercyclical response, underlying structural weakness have transformed a temporary shock into persistent vulnerability. The country's primary revenue base, the hydrocarbon sector [2], faces a dual threat: secularly declining production volumes [3] coupled with volatile energy prices trending downward from 2022 through 2025. This structural decline directly translates into dwindling government tax revenues, creating an enduring fiscal shortfall. Without drastic expenditure cuts, the government is structurally condemned to operate persistent fiscal deficits, which in turn fuel further debt accumulation.

Although technical analytical framework exists; such as the International Monetary Fund's (IMF's) Debt Sustainability Analysis (DSA) for market-access and low-income countries [4], [5]; their implementation requires advanced mathematical skills and extensive datasets that are often unavailable in developing countries. The established literature excels at identifying vulnerability under current policies but offers limited actionable guidance on the specific fiscal burden of adjustment required to achieve predefined sustainability targets. This creates a significant operational gap between diagnostic tools and practical fiscal planning: policymakers recognize the existence of a sustainability problem but lack sufficient datasets and technical skills to translate debt targets into concrete annual fiscal surplus requirements.

This paper addresses this gap by proposing a Progressive Web Application (PWA) for computing the fiscal adjustment path required to achieve a target debt-to-GDP ratio. Thus, the objective of this study is to propose and design a computational framework for a PWA to derive the fiscal surplus required to bring the debt-to-GDP ratio to a sustainable level of 60% of GDP in 10 years. T&T, is selected to empirically apply such framework. As such, the corresponding research question is "What is the design of a PWA for computing the requisite fiscal surplus to reduce T&T's public debt-to-GDP ratio to 60% within a decade?"

The core computation task integrates a multi-step forecasting and amortization framework. It begins with a non-linear time-series forecast of nominal GDP, proceeds to calculate a target future debt stock, and finally computes the present value of the annuity payments required to amortize the excess target debt over the decade. The result translates an abstract sustainability goal into a tangible annual fiscal target.

This paper presents the complete econometric model for estimating the required fiscal adjustment path and provides the design methodology for embedding this model within a PWA. It does not deliver a fully coded application, thereby allowing developers to adapt the proven analytical core to their analytical core to their preferred software architecture and user interface. As such, the contribution is the reusable fiscal model with is adaptable to PWA implementation.

The rest of this study is structured as follows. Section 2 presents a literature review, situating this work within the domains of debt sustainability analysis, and PWAs. Section 3 details the methodology, including the framework for the debt sustainability analysis, the data, and the system architecture of the developed application. Section 4 reports the results of the debt sustainability analysis. Section 5 discusses the process for building the PWA, and the tool's limitations. Section 6 summarizes the contributions made. Finally, Section 7 concludes this study.

2. Literature Review

2.1. Debt Sustainability

Debt sustainability gained prominence following the Latin American debt crises of the 1980s and the subsequent fiscal challenges in advanced economies, evolving into a critical tool for policymakers and international institutions [6]-[8].

At its core, debt sustainability is about the feasibility of debt repayment. A government's fiscal trajectory is considered sustainable if it can continue servicing its debt indefinitely under current policies, or with feasible and desirable policy adjustments, without triggering a crisis. The emphasis on "feasible and desirable" is crucial; it rules out strategies like default, disorderly restructuring, or the imposition of austerity so severe it cripples growth or social stability. Sustainability is not a fixed point but a zone of stability, contingent on economic expectations, investor confidence, and the institutional capacity to generate future primary surpluses (public revenues minus non-interest public expenditure).

The challenge, as illustrated by cases like Japan (with debt over 200% of GDP) and Ukraine (which defaulted with debt at 30% of GDP in 1998) [9], [10]. Although, a debt-to-GDP ratio of 60% is now considered as the acceptable sustainable threshold [11]-[13]. Debt sustainability is a function of an interplay between a country's specific economic structure, debt characteristics, and the global financial environment [14]. A pragmatic approach to sustainability therefore moves beyond rigid metrics, incorporating judgement of repayment capacity, a country's economic growth prospects, institutional strength, and vulnerability to shocks [15].

Solvency is the long-term necessary condition for debt sustainability [16]. A government is solvent if the present value of its future primary budget surpluses is at least equal to the current market value of its debt [17]. In essence, solvency asks: can the government repay its debt in the long-run?

The foundational arithmetic of debt solvency rests on the logic about the relationship between a government's debt, its budget, and the size of its economy [18]. The core idea is that the change in a country's debt burden; measured as debt-to-GDP; is driven by the interplay of these fiscal conditions.

The difference between the interest rate (r) the government pays on its debt, and its country's economic growth (g) is very important fiscal condition [19]-[21]. If the government's borrowing cost outpaces the economy's capacity to grow, the debt burden has a tendency to be problematic. Moreover, it can put the country on a trajectory where high debt or increasing debt may correspond with a decline in a country's economic grow-th. Conversely, if the economy's growth rate is higher than the interest rate, this creates a favorable condition that naturally helps to shrink the debt burden over time.

Another important fiscal condition is the primary fiscal balance. This is, the government's budget position excluding interest payments [22]. This is the direct, discretionary level of fiscal policy. A primary surplus (public revenues greater than non-interest public expenditure) directly pulls down the debt stock, while a primary deficit adds to it.

Solvency, therefore, is not about having no debt. It is about the government's capacity to manage this dynamic equation [23]. A sustainable path is one where, over the long-term, the combination of the primary balance and the interest-growth differential ensures that the debt-to-GDP ratio does not embark on an ever-accelerating, explosive trajectory that would necessitate a crisis response. The government must generate enough fiscal effort to offset any unfavorable differential between interest rates and growth. Or it must rely on a favorable differential to compensate for periods of primary deficit. The long-run solvency condition is essentially a promise that the present value of all future primary surpluses will be sufficient to cover the existing debt.

Solvency is encapsulated in the Intertemporal Government Budget Constraint (IGBC) and a No-Ponzi-Game Condition (NPGC) [20]. The IGBC stats that initial debt must equal the present value of future primary surpluses. The NPGC rules out financing debt forever by issuing new debt to pay interest. However, the IGBC's practical application is limited. It assumes perfect market access at a known discount rate (r) and requires projections over an infinite horizon, making it an unreliable operational tool. Furthermore, in a stochastic world with risk-averse investors, the correct discount factor for future surpluses is not the government's borrowing rate, but a stochastic discount factor reflecting the marginal utility of consumption. Since governments often run deficits in bad times (procyclical fiscal policy), the stream of surpluses provides poor insurance value, potentially raising the required risk-adjusted discount rate and the tightening the solvency constraint.

Therefore, while solvency provides the theoretical anchor for debt sustainability, it is an incomplete guide for real-time policy.

Liquidity addresses the timing and smoothness of debt service [19], [20]. A government can be solvent (its long-term finances exceed debt) but illiquid if it lacks sufficient funding to meet short-term debt service obligations [24]. Liquidity crises are about rollover risk; the inability to refinance maturing debt at acceptable interest rates, potentially leading to a default that was not fundamentally inevitable.

Liquidity risk is influenced by several factors. One factor is the debt structure. High shares of short-term debt or foreign-currency debt increase refinancing frequency and vulnerability to exchange rate swings. Another factor is market sentiment. Self-fulfilling panics can occur if investors, fearing default, demand prohibitively high interest rates, making debt servicing unsustainable and triggering the very default they feared.

Another factor is gross financing needs. This is a key liquidity metric, representing the total financing required in a given year (the overall deficit plus the amortization of maturing debt). High gross financing needs forces the government to be a frequent and large presence in markets, increasing exposure to sentiment shifts.

Another factor is the buffer of financial assets. A high level of foreign exchange reserves or other liquid assets can cushion against temporary market closures.

For many emerging market and developing economies, liquidity is often the binding constraint [25]. They may have moderate debt ratios but face volatile capital flows, "sudden stops," and shallow domestic debt markets. The distinction between illiquidity and insolvency is often blurred at the onset of a crisis, as a liquidity shock can itself undermine solvency by drastically altering the $r - g$ dynamic.

Recognizing the limitations of pure solvency arithmetic and the importance of liquidity, the IMF developed Debt Sustainability Analysis (DSA) frameworks tailored to different country circumstances. The IMF employs two distinct frameworks, namely the Market-Access Countries (MACs), and the Low-Income Countries (LICs) [4].

In the MAC DSA, this focus is on total public debt. Analysis centers on projecting the debt-to-GDP ratio under a baseline macroeconomic scenario and conducting stress tests. The assessment is guided by debt thresholds, which signal levels beyond with the risk of fiscal distress historically increases [5].

In the LIC DSA, the focus is on the present value of external public and publicly guaranteed debt. The present value metric, by discounting future repayments at a market-related rate, better reflects the true burden of soft loans that the nominal debt stock. The framework compares present value debt ratios to GDP, exports, and revenue, using thresholds that account for the quality of a country's policies and institutions [5].

While the established literature on debt sustainability provides a framework to analyze debt, a significant operational gap remains. The existing frameworks excel at identifying vulnerability under current policies, but they offer less actionable guidance on the specific fiscal burden of adjustment required to achieve a predefined sustainability target.

This study contributes to the literature by moving from diagnosis to prescription. In other words, this study calculates the explicit annual amortization costs; the fiscal surplus needed to reduce the debt to a benchmark sustainable level over a decade. This changes the abstract notion of a "feasible adjustment" into a tangible, time-bound fiscal cost, thereby bridging the theoretical concept of solvency with an actionable fiscal target.

The methodology for such assessment is expressed in Section 3, and the corresponding results are displayed in Section 4.

2.2. Progressive Web Apps

The concept of PWA emerged from a recognition of the growing divergence between native mobile applications; which offered rich, reliable experiences; and the mobile web, which offered a wide reach but suffered from performance and capability limitations. PWAs present a technical model for web development that uses modern web capabilities to deliver app-like user experiences. PWAs are merely standard web-based programs that mimic native apps by utilizing modern web browsing capabilities. PWA offers features like background synchronization, offline support, and home screen installation for a variety of devices in addition to enabling cross-platform development across websites. Service worker, application shell, and web app manifest are the technology concepts introduced by PWA that enable it to come together [26].

Service workers in a PWA function as a proxy between the browser and the network, operating in the background to enable essential features like offline access, push notifications, and background sync. They are essential for providing native-app-like speed, enabling PWAs to safely operate without the internet and load instantly from cache [27].

The Application Shell (App Shell) Model in a PWA is an architectural strategy that emphasizes loading a minimum user interface instantaneously, giving the user a quick, dependable, and native-app-like experience. The fundamental HTML, CSS, and JavaScript that drive the user interface are provided by the app shell, which acts as the application's 'skeleton' or basic framework. After then, a service worker caches it offline so that this shell can be loaded promptly from the local cache on subsequent visits, eliminating the need to constantly fetch it from the network.

The web app manifest (a JSON file) instructs browsers how the PWA should appear and function when loaded on a user's device. It gives the operating system the necessary metadata to make an installable shortcut and manage how the application appears in the operating system shell instead of a browser tab [28].

PWAs have seen successful adoption across diverse sectors. PWAs have been effectively used by companies in a variety of industries, including e-commerce, tourism, food and beverage, media and publishing, social media, and more, to increase user engagement.

There is academic work exploring the use of PWAs in health care [29], education [30], finance and other industries [31]. This study proposes the extension of this trajectory into the domain of economic policy and econometric modelling. It represents a novel application of the PWA paradigm to create a tool to provide the much needed economic analysis for policy makers.

The next section outlines the methodology for this study.

3. Methodology

3.1. Fiscal Surplus Required to Stabilize the Debt

The evaluation of the sustainability of a country's public debt entails estimating the fiscal surplus required to bring a country's debt to a sustainable level [1]. The benchmark sustainable level for debt is 60% of GDP. This can be achieved within a decade. Therefore, the analysis can be performed to determine the fiscal surplus that is required to reduce the debt to GDP ratio within 10 years.

This analysis can be performed via several steps.

- Step 1 – Forecast the GDP for the respective country. A non-linear regression model can be used to provide a 10- step ahead out of sample forecast for the GDP. This will be the 10 year forecast. The Manifold-Constrained Hyper-Connection is used to provide the forecast.
- Step 2- Compute the debt at the end of the period. Compute the total debt stock that is equal to 60% of the GDP.
- Step 3 – Estimate the difference between the current and the target debt. This involves estimating the difference between the current total debt stock in 2024 and the forecasted debt stock in 2034. The year 2024 is selected as it is the most recent year with debt data for T&T. This difference will be debt that the government(s) desires to eliminate. For the purposes of this analysis, let this debt be referred to as "targeted debt".
- Step 4 – Estimate the amortization of the debt. Consistent with the debt sustainability analysis framework of the IMF, the targeted debt should be amortized. This technique will estimate the present value of repaying the debt over 10 years.

Notably, the amortization payment for each year will be the cost of debt service to the government. The government will have to run a fiscal surplus equal to this estimated cost in order to repay this debt over 10 years. This debt service will be the cost to the people of the country, since it is the tax that is paid by the people that will be used to service the debt.

So, for the 10-year period, if the government decides to repay its debt to reduce the debt to GDP ratio to 60% of GDP, it must operate a fiscal surplus that is equal to the results of the aforementioned estimation.

3.2. Manifold-Constrained Hyper-Connection

3.2.1. Motivation and Context

Deep learning has gradually advanced through new activation functions or loss formulations than through architectural innovations governing information flow.

Residual connections, dense connections, attention mechanisms, and hyper-networks have all demonstration how information propagates through a network is often more important than the specific nonlinearities applied at each layer. These architectures have enabled unprecedented depth, improved optimization stability, and stronger generalization. However, they share a common and largely unexamined assumption; that intermediate representations exist in an unconstrained Euclidean space and may be combined freely using linear or nonlinear operators without regard to their underlying geometric structure.

This assumption is increasingly at odds with both empirical evidence and theoretical insights. Learned representations in deep networks concentrate near low-dimensional manifolds embedded in high-dimensional ambient spaces. These manifolds reflect intrinsic factors of variation in the data; such as pose, lighting, syntax, or latent economic drivers; and tend to be smooth, structured, and locally low-rank. Yet most architectures allow information to flow in directions orthogonal to these manifolds, effectively wasting representation capacity and introducing instability. As connectivity becomes richer, especially through dense or hyper-connections, the risk of drifting off-manifold increases, leading to redundancy, overfitting, and optimization difficulties.

Manifold-Constrained Hyper-Connections (mHC) [32] are motivated by the observation that flexible connectivity and geometric discipline should not be treated as competing objectives. Instead, the authors argue that connectivity should be maximally expressive subject to geometric constraints. In other words, networks should be free to learn how layer interact, but those interactions should be restricted to directions that respect the intrinsic structure of learned representations. mHC formalizes this idea by embedding manifold constraints directly into the definition of hyper-connections, thereby aligning architectural flexibility with representation geometry.

3.2.2. From Skip Connections to Hyper-Connections

To appreciate the contribution of mHC, it is useful to trace the evolution of inter-layer connectivity in deep networks. Consider a standard feed-forward network with representations $h_1 \in \mathbb{R}^d$ and layer l. The forward pass is given by:

$$h_{l+1} = f_l(h_l) \quad (1)$$

where f_1 is a learned nonlinear transformation. This is strictly sequential structure suffers from vanishing gradients and poor feature reuse. Residual networks address this by introducing identify skip connections.

$$h_{l+1} = f_l(h_l) + h_l \quad (2)$$

which can be interpreted as learning residual perturbations around an identity map. which can be interpreted as learning residual perturbations around an identity map. DenseNets generalize this idea further by concatenating all previous representations.

$$h_{l+1} = f_l([h_1, h_2, \dots, h_l]) \quad (3)$$

thereby maximizing feature reuse and improving gradient flow.

While effective, both residual and dense connections rely on fixed, hand-designed connection rules. Hyper-connections remove this restriction by allowing the network to learn how information from earlier layers should influence later ones. A generic hyper-connected layer can be expressed as:

$$h_{l+1} = f_l\big(h_{l,}, \emptyset_l[h_1, h_2, \dots, h_{l-1}]\big) \quad (4)$$

where $\emptyset_l$ is a learned function, often implemented via a hyper-network that dynamically generates connection weights or modulation signals. This formulation greatly expands expressive power, but it also introduced new challenges. Since $\emptyset_l$ operates in the full ambient space $\mathbb{R}^d$, it may introduce arbitrary interactions that ignore the structure of learned features. As depth and connectivity increase, this unconstrained flexibility can destabilize training and inflate the effective hypothesis space.

mHC can be seen as a response to this problem. It retains the expressiveness of hyper-connections while imposing principled constraints on where and how information may flow.

3.2.3. Core Idea of Manifold-Constrained Hyper-Connections

The core idea of mHC is simple. Hyper-connections should operation on manifolds rather than in unconstrained Euclidean space. Instead of allowing arbitrary transformations between layers, mHC restricts inter-layer communication to a learned low-dimensional manifold that captures the intrinsic structure of representations.

Intuitively, one may think of each layer's representation as a point on a surface embedded in high-dimensional space. Standard hyper-connections allow the network to jump arbitrarily through space, potentially leaving the surface altogether. mHC, by contrast, allows the network to move freely along the surface, but not off it.

This idea is closely related to concepts in differential geometry and dynamical systems. If representations evolve along a manifold, then learning can be interpreted as navigating a structured state space rather than exploring an unstructured vector space. Hyper-connections then correspond to learned vector fields on the manifold, guiding representation flow in a stable and interpretable manner.

3.2.4. Representation Manifolds in Deep Networks

Given a dataset $\{x_i\}_{i=1}^{N} \subset \mathbb{R}^d$, deep networks tend to map inputs into representations whose intrinsic dimensionality is smaller than d. One may assume that for each layer l, there exists a manifold $\mathcal{M}_l \subset \mathbb{R}^d$ of dimension $k \ll d$ such that:

$$h_{l,} \in \mathcal{M}_l \text{ for most training scripts} \quad (5)$$

These manifolds are not fixed; they evolve during training and differ across layers. Early layers may capture local features with relatively high intrinsic dimension, while deeper layers often collapse variability into highly structured, task-relevant representations.

Ignoring this structure has consequences. When networks operate in full ambient space, they waste capacity modeling directions orthogonal to $\mathcal{M}_l$ and allow noise to propagate along meaningless axes. In contrast, explicitly modeling $\mathcal{M}_l$ allows the network to focus computation on meaningful degrees of freedom. mHC using this insight by embedding manifold structure directly into the connectivity pattern rather than treating it as an emergent property discovered post hoc.

3.2.5. Formal Definition of mHC

mHC formalizes manifold structure by introducing a latent coordinate system for each layer. Specifically, the representation $h_{l,} \in \mathbb{R}^d$ is parameterized as

$$h_{l,} = g_l(z_l), \qquad z_l \in \mathbb{R}^d, \qquad k \ll d \quad (6)$$

where $g_l: \mathbb{R}^k \rightarrow \mathbb{R}^d$ is a learned embedding function defining the manifold $\mathcal{M}_l = \{g_l(z): z \in \mathbb{R}^d\}$. The latent variable z_l represents intrinsic coordinates on the manifold.

Hyper-connections are then defined in latent space rather than ambient space. A generic mHC update takes the form:

$$z_{l+1} = \psi_l(z_l, z_1, \dots, z_{l-1}) \quad (7)$$

where ψ_l is a learned hyper-connection function operating in $\mathbb{R}^k$. The ambient-space representation is recovered via:

$$h_{l+1} = g_{l+1}(z_{l+1}) \quad (8)$$

This formulation ensures that all representations remain on their respective manifolds by construction. Unlike unconstrained hyper-connections, which may generate representations far from any meaningful data structure, mHC restricts updates to directions that can be expressed through the learned embedding g_l.

3.2.6. Manifold Constraints

Enforcing manifold structure requires additional constraints to ensure that representations do not drift off-manifold during training. mHC employs a combination of explicit and implicit mechanisms to achieve this. One approach is explicit projection. Given a provisional representation $\tilde{h}_l$, the model applies a projection operator $\Pi_{\mathcal{M}_l}$ such that:

$$h_l = \Pi_{\mathcal{M}_l}(\tilde{h}_l) = g_l(g_l^{-1}(\tilde{h}_l)) \quad (9)$$

where g_l^{-1} is an approximate inverse or encoder. This ensures that h_l lies on $\mathcal{M}_l$ even if intermediate computations deviate slightly.

Another mechanism is manifold regularization, which penalizes deviations from the manifold through an auxiliary loss term,

$$L_{manifold} = \sum_l \|h_l - g_l(g_l^{-1}(h_l))\|^2 \quad (10)$$

This encourages representations to be reconstructible from their latent coordinates, effectively constraining them to remain close the learned manifold.

Additional constraints may penalize excessive curvature or rapid changes in local geometry, for example by regularizing the Jacobian of g_l.

$$L_{smooth} = \sum_l \|\nabla_z g_l(z_l)\|^2 \quad (11)$$

Together, these constraints ensure that the manifold is smooth, low-dimensional, and stable under hyper-connected updates.

3.2.7. Overall Model

The full mHC model integrates task learning, hyper-connections, and manifold constraints into a single objective. Let L_{task} denote the primary loss. Therefore, the overall optimization problem is

$$L = L_{task} + \lambda_1 L_{manifold} + \lambda_2 L_{smooth} \quad (12)$$

where λ_1 and λ_2 control the strength of geometric regularization. Training proceeds end-to-end, learning both the task-specific mappings and the manifold structure that constrains connectivity.

Conceptually, mHC transforms the deep network into a dynamical system on a learned manifold. Figure 1 illustrates the mHC model.

3.3. Data

This study centers on two main macroeconomic variables for T&T, namely GDP, and debt. The real GDP serves as the denominator for the debt ratio and the base for economic scaling. Annual historical GDP in 2018 constant prices (in US currency) from 1990 to 2024 is sourced from the Economic Commission for Latin America and the Caribbean (ECLAC/CEPALSTAT) database, reputable source for regional economic data.

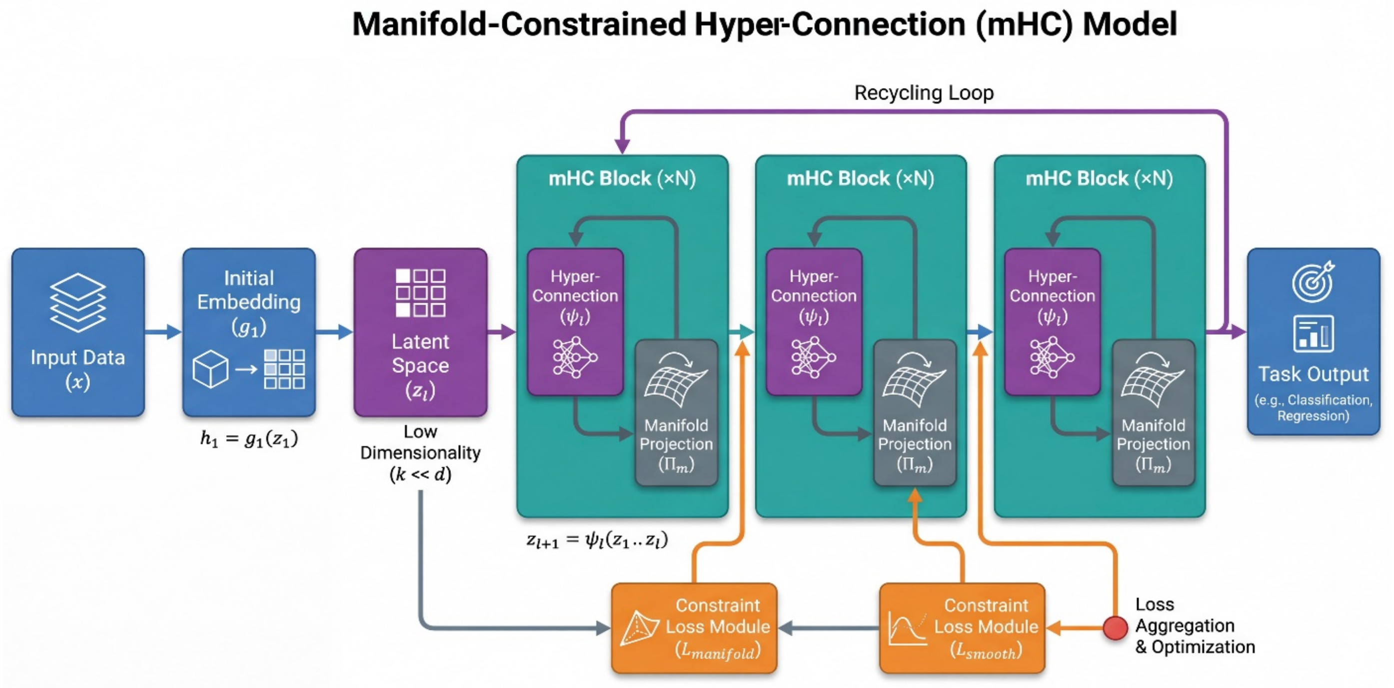

Figure 1. mHC Model.

The total debt-to-GDP ratio is other key variables. The 2024 year-end value of 75.6% is taken as the starting point, based on the latest official estimates from the Central Bank of Trinidad and Tobago (CBTT). The target sustainable ratio is set at 60%.

3.4. Data Processing Framework

The analytical framework involves several steps.

- Step 1: Stationarity, Linearity and Normality Tests. First, the data is tested for stationarity, linearity, and normality. These tests are important since if the data is non-linear and not normally distributed, then a traditional linear region model based on the assumption of normality is not appropriate for modelling. This justifies the application of a non-linear regression framework.
- Step 2: The GDP series is regressed with a model and a 10-steps ahead out-of-sample forecast is generated. This can be done with the mHC model.
- Step 3: Derivation of debt Stocks. Using the GDP forecasts, the absolute debt stocks are calculated. That include the 2024 debt stock, calculated as the GDP (2024) * 0.756. Additionally, the target debt stock (2034) is calculated as the forecasted GDP (2034) * 0.60.
- Step 4: Calculate the fiscal adjustment requirement. The required debt reduction is the difference between the estimated 2024 debt stock and the 2034 target debt stock. This future value represents the total nominal amount of excess debt that must be amortized over the 10-year period.
- Step 5: Amortization of the debt. To translate the total debt reduction into an annual fiscal surplus path, a standard present value amortization is applied. A discount rate of 5% per annum is applied. The payments are modelled as annual, reflecting targeted primary fiscal surpluses. This is consistent with the principle that “the government must generate enough fiscal effort to offset any unfavorable differential.

3.5. Cross Comparison

For cross comparison, the mHC model will be compared to the Autoregressive Integrated Moving Average (ARIMA) model. The ARIMA model is a linear time series framework used for modeling and forecasting univariate data by capturing dependence structures through 3 components, namely: i) autoregression; ii) the order of integration; and iii) moving average. The autoregressive component models the current value of a series as a function of its past values. The order of integration component applies differencing to render a trend non-stationary series as stationary. The moving average component accounts for series correlation in the error by incorporating past forecast errors. As such, an ARIMA(p,d,q) specification is defined by the number of autoregressive lags (p), the differencing required (d), and the number of moving average terms (q).

3.6. Diagnostics

Several diagnostics and error metrics will be applied. Mean Squared Error (MSE):

$$MSE = \frac{1}{n}\sum_{i=1}^{n}(y_i - \hat{y}_i)^2 \quad (13)$$

where y_i is the actual value, $\hat{y}_i$is the predicted value, and n is the number of observations.

Root Mean Squared Error (RMSE):

$$RMSE = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(y_i - \hat{y}_i)^2} \quad (14)$$

This is the square root of the MSE, bringing units back to the original scale.

Mean Absolute Percentage Error (MAPE):

$$MAPE = \frac{100\%}{n}\sum_{i=1}^{n}\left|\frac{y_i - \hat{y}_i}{y_i}\right| \quad (15)$$

This represents the average absolute percentage difference between predicted and actual values.

Mean Absolute Error (MAE):

$$MAE = \frac{1}{n}\sum_{i=1}^{n}|y_i - \hat{y}_i| \quad (16)$$

This represents the average magnitude of errors. Residual plots will be applied to assess the statistical adequacy of the mHC model. Specifically, the Quantile-Quantile (Q-Q) plot will be used to evaluate the extent in which the residuals are normal and white noise. The histogram confirms the symmetry of the distribution of the residuals around 0. The autocorrelation function tests for independence in the errors.

The peak demand performance evaluation will be used to examine the model's robustness during high-value periods. The scatter plot applied in the peak demand performance evaluation can determine systematic biases. However, error metrics (MAE, RMSE, and MAPE) show performance degradation during peak versus non-peak conditions. The actual versus predicted GDP will show a comparison between observed and modeled values to assess the overall predictive accuracy.

Temporal validation will also be implemented, evaluating the model's generalization across time periods by splitting data into sequential windows. This approach assesses forecasting performance and detects overfitting by testing performance consistency on unseen future segments.

SHapley Additive exPlanations (SHAP) values are also applied. SHAP values is a framework from game theory that is used for interpreting machine learning predictions by attributing each feature's exact contribution to individual outputs. They break down a prediction to show how much each input moves the result away from the average prediction, allowing for global and local interpretability.

After the aforementioned metrics are applied, the mHC's performance will be assessed by comparing it to the ARIMA model. This is done through the use of the Wilcoxon framework. The Wilcoxon Signed-Rank test is a non-parametric paired test to determine whether the median of paired differences is zero. It requires paired observations, the symmetry assumption on distribution of differences, ranking of absolute differences, and testing whether the median difference is not equal to 0.

However, this study applies the Wilcoxon Signed-Rank Test on Multiple Metrics to evaluate whether the performance differences between the mHC and ARIMA models are statistically significant across multiple dimensions of forecast behavior.

The methodology operates be constructing paired comparisons across 4 different metrics that capture the structural characteristics differentiating the models.

- First, a non-linearity measure was computed by fitting linear trends to each model's 10 forecasted values and calculating the absolute deviations from these trends. This metric quantifies the extent to which each model captures non-linear dynamics.
- Second, forecast stability was assessed through step-to-step changes, capturing volatility patterns.
- Third, bootstrap resampling was applied to each model's forecast to approximate the empirical sampling distribution of the mean forecast. This approach tests whether the models' forecasts come from fundamentally different populations.
- Fourth, volatility evaluation was analyzed using rolling window standard deviations across the forecast horizons, capturing how uncertainty accumulates different between the models.

For each metric, the test ranks the absolute differences between paired observations, assigns signs based on whether the mHC or ARIMA produces larger values, and calculates a test statistic based on the sum of positive ranks. The resulting p-values indicate whether the observed difference across each dimension are statistically significant, or attributable to random variation.

The next section presents the results of the empirical analysis. It shows the fiscal surplus required to stabilize T&T's debt.

4. Results

4.1. Pretesting Results

Before undertaking the forecasts, some pretests are performed. The pretesting results are displayed in Table 1.

As can be seen from Table 1, the ADF and KPSS tests both provide consistent evidence that the GDP time series is non-stationary. The high p-value (0.5107) in the ADF test shows that the null hypothesis of a unit root cannot be rejected, and the test statistic (−1.5458) is far

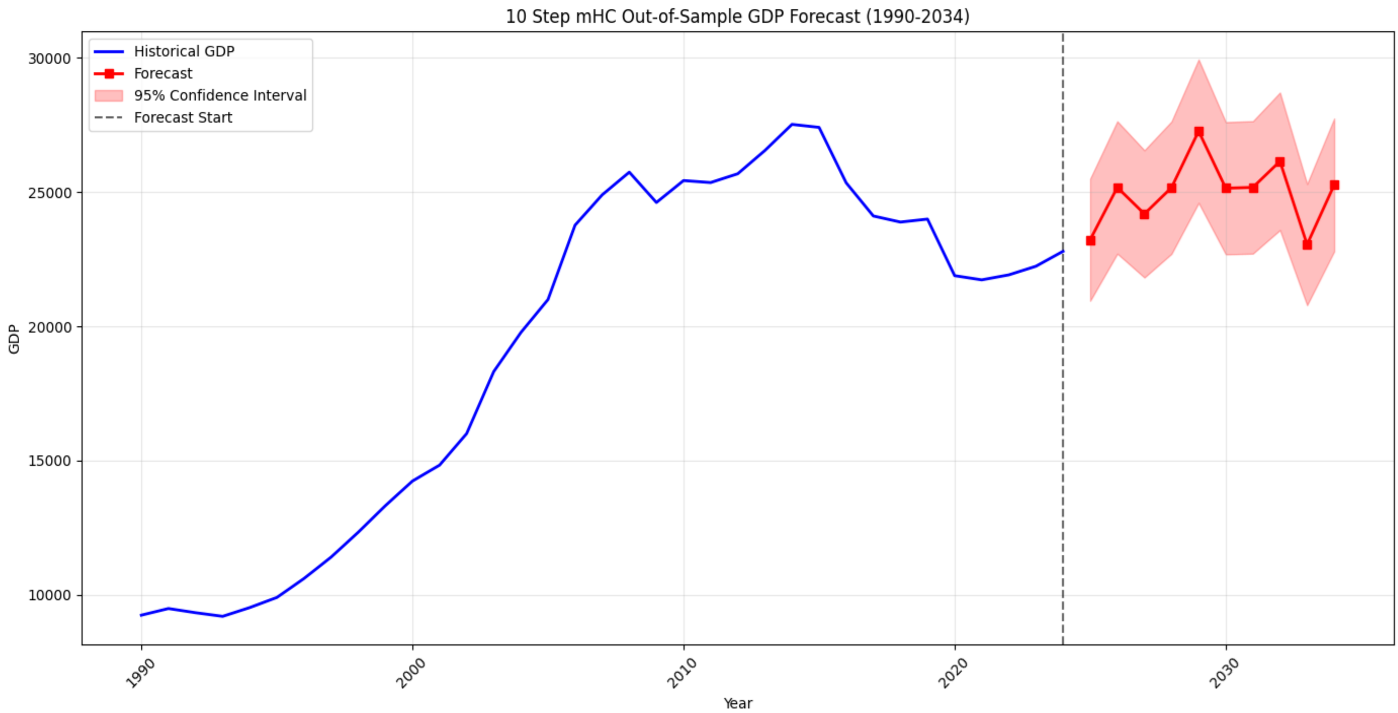

Figure 2. mHC Forecasts on T&T's GDP.

Table 1. Pretesting Results.

Test	Null Hypothesis	Test Statistic	P-value	Critical Values	Conclusion
Augmented Dickey-Fuller (ADF)	Series has a unit root (non-stationarity)	-1.5458	0.5107	1%: -3.646 5%: -2.954 10%: -2.616	Fail to reject H_0 Non-stationary
KPSS	Series is stationary	0.6325	0.0197	10%: 0.347 5%: 0.463 2.5%: 0.574 1%: 0.739	Reject H_0 Non-stationary
Rainbow Test for Linearity	Model is linear over the range	7.8934	0.0001		Reject H_0 Model is non-linear
Shapiro-Wilk Test	Data follows a normal distribution	0.8648	0.0005		Reject H_0 Data is not normally distributed

above the critical values at all traditional significance thresholds. This suggests that instead of oscillating around a steady mean, the series shows persistent stochastic trends. This conclusion is supported by the KPSS test, which rejects the null hypothesis of stationarity with a p-value of 0.0197 and a test statistic of 0.6325 that exceeds the 5% and 2.5% critical values. When combined, these tests show that the GDP series is essentially non-stationary, a feature common to macroeconomic aggregates and problematic for conventional linear time-series models that depend on stationarity assumptions.

The Rainbow test for linearity reveals that the underlying data-generating process is not adequately represented by a linear specification. The null hypothesis that the model is linear over the whole range of the data is clearly rejected due to the large test statistic (7.8934) and small p-value (0.0001). This finding implies that GDP dynamics differ across levels or regimes.

The Shapiro-Wilk test offers information about the distributional characteristics of the series (or residuals). The test reveals a significant deviation from normality. The null hypothesis of normality is implied by the low-test statistic (0.8648) and p-value (0.0005), which suggest skewness, heavy tails, or other higher-order distributional characteristics.

Collectively, these diagnostics provide a clear picture of a time series that is non-stationary, nonlinear, and non-Gaussian. This provides compelling modeling justification for the mHC model, which does not depend on stationarity, linearity, or normality assumptions. As such the mHC model is run, and its results are presented in the next subsection.

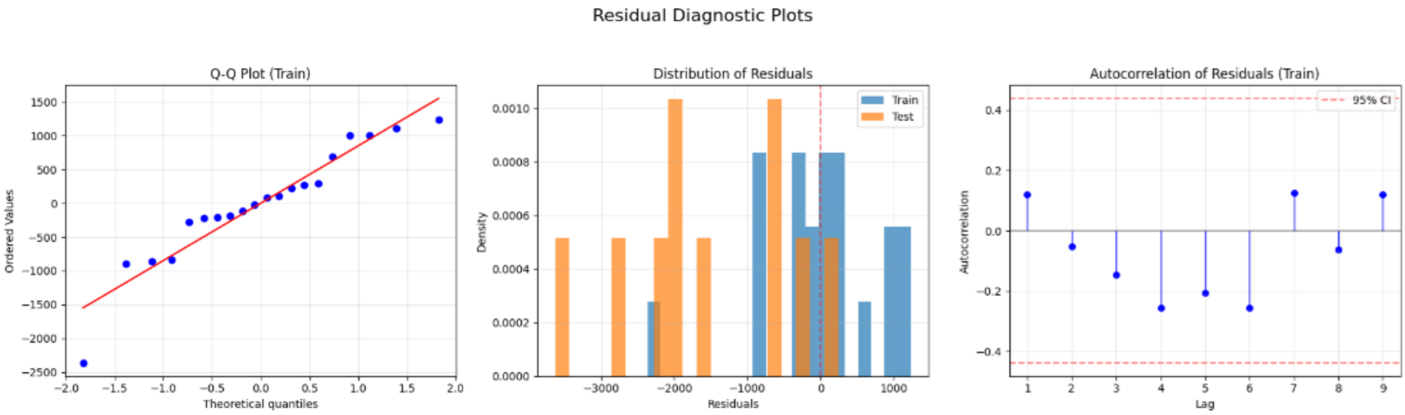

Figure 3. Residual Plots.

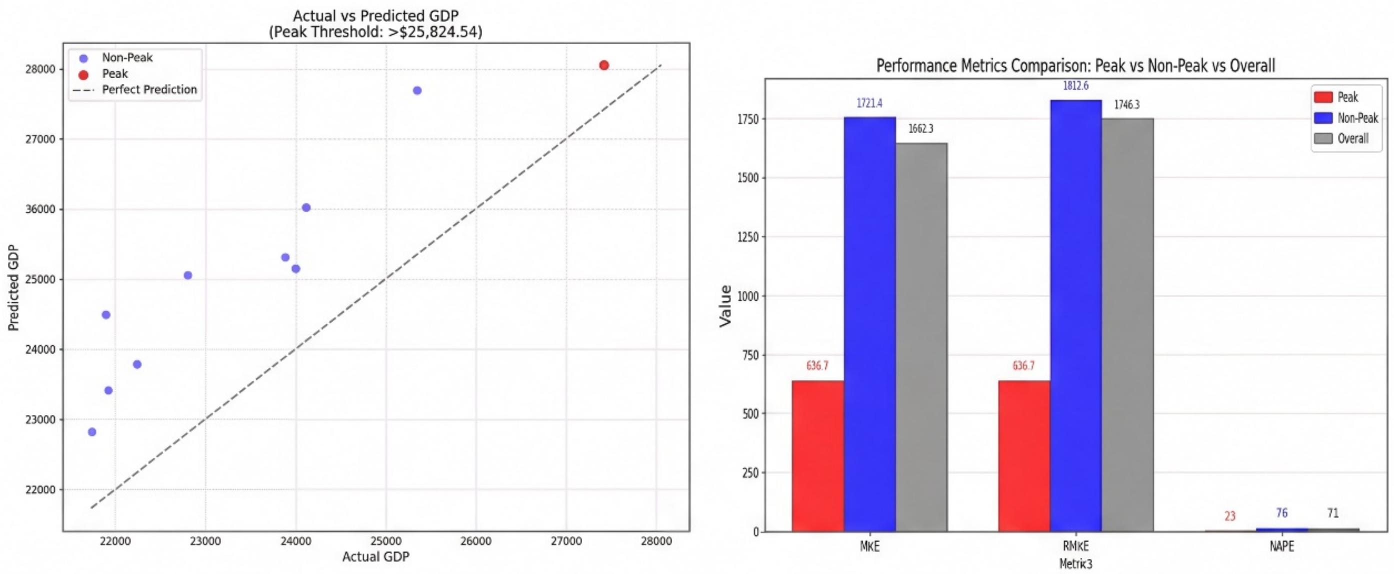

Figure 4. Peak Demand Performance Evaluation.

4.2. mHC Forecasting Results

The mHC model is applied to forecast T&T's GDP. The forecast is displayed in Figure 2. Figure 2 showed the full data as well as the 10-step ahead out-of-sample forecast from the mHC model. The out-of-sample forecasts were as follows:

- Step 1: 23,225.4089
- Step 2: 25,174.7110
- Step 3: 24,189.1947
- Step 4: 25,168.5730
- Step 5: 27,271.8003
- Step 6: 25,146.5851
- Step 7: 25,176.5821
- Step 8: 26,149.8719
- Step 9: 23,046.6941
- Step 10: 25,268.8501

Diagnostics are applied to the mHC to determine model authenticity. The first diagnostic results considered are the residual plots, displayed in Figure 3.

As can be seen in Figure 3, the residual diagnostic plots provide evidence of the mHC model's statistical adequacy for GDP forecasting. The Q-Q plot shows that the residuals follows approximate white noise. The histogram of the residuals display a relative symmetric, bell-shaped distribution centered at 0 for both training and test sets. The autocorrelation function plot displays all the lagged correlations falling within the 95% confidence bands, indicating no significant serial correlations in the residuals.

As seen in Figure 4, the peak demand performance evaluation reveals that the mHC model demonstrates robustness in forecasting GDP during high-value periods. As illustrated in the scatter plot of actual versus predicted values, both peak and non-peak observations cluster relatively around the perfect prediction line, indicating no systematic bias in the model's estimations. The MAE, the RMSE, and the MAPE, all show that the performance of the model displays modest degradation during peak demand thresholds relative to the non-peak thresholds.

Figure 5 displays the fit of the mHC model, relative to the actual data. The model displays a relatively good alignment with the actual GDP throughout the period, suggesting a good fit. The corresponding error metrics were as follows.

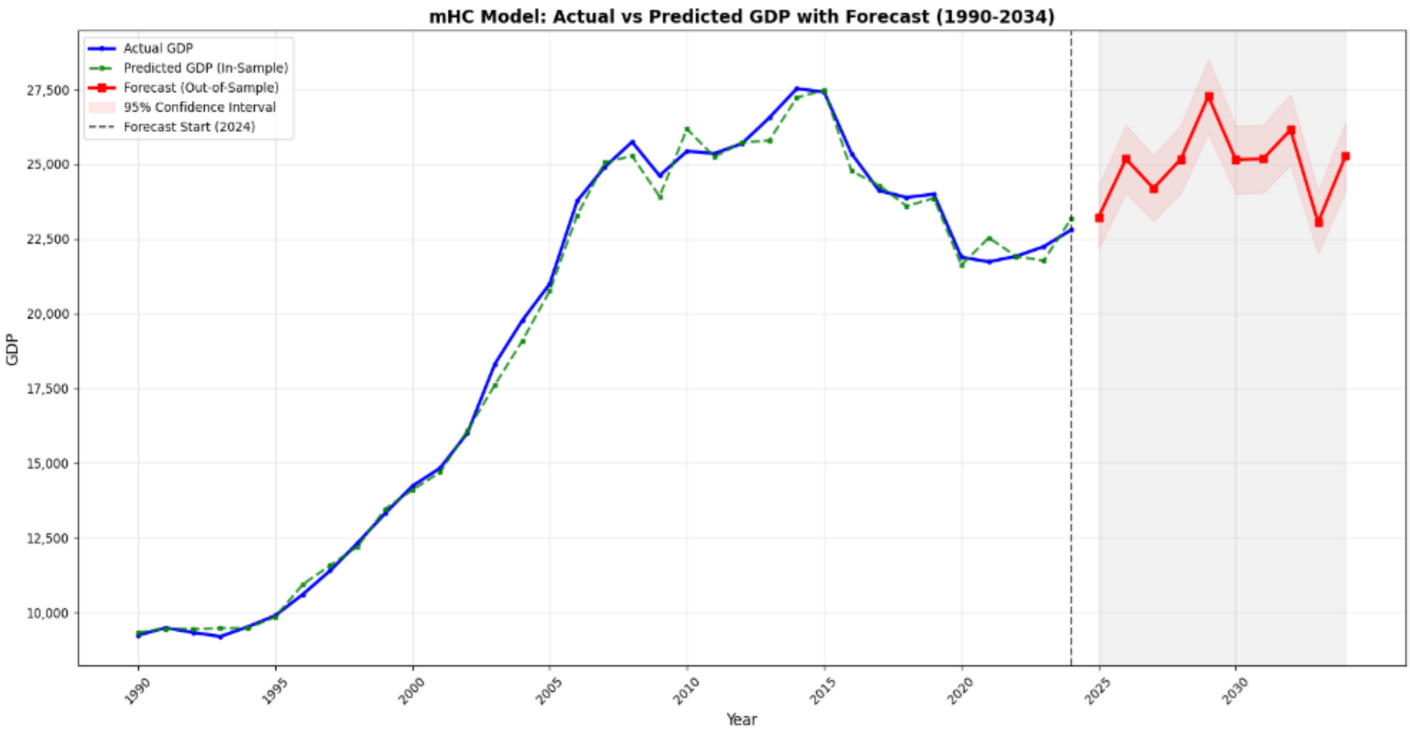

Figure 5. Actual vs Predicted GDP with Forecast.

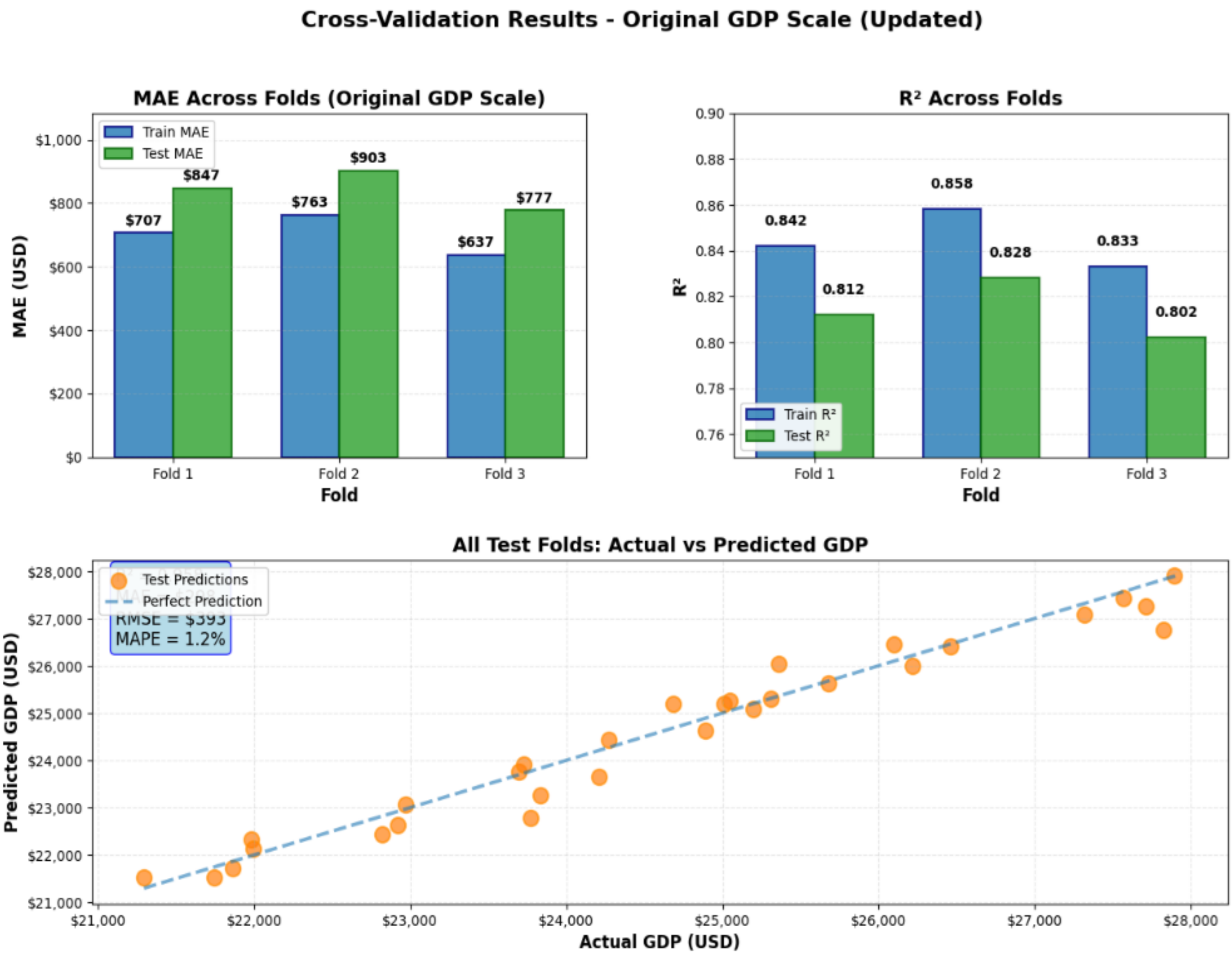

Figure 6. Cross Validation Results.

Table 2. Forecasted Public Revenue and Expenditure for T&T with the mHC.

Date	Central Government Total Revenue (TTD million)	Central Government Total Expenditure (TTD million)	Fiscal Balance (TTD million)	Fiscal Balance (USD million)
31/12/2025	47,692.64	63,156.33	-15463.7	-2379.03
31/12/2026	51,601.88	64,992.30	-13390.4	-2060.06
31/12/2027	45,853.04	66,136.65	-20283.6	-3120.56
31/12/2028	51,823.86	66,745.40	-14921.5	-2295.62
31/12/2029	49,134.54	66,187.91	-17053.4	-2623.6
31/12/2030	48,160.75	65,712.33	-17551.6	-2700.24
31/12/2031	53,402.46	67,125.49	-13723	-2111.24
31/12/2032	43,028.92	66,550.59	-23521.7	-3618.72
31/12/2033	55,065.65	66,030.73	-10965.1	-1686.94
31/12/2034	50,379.80	67,184.12	-16804.3	-2585.28

Table 3. Amortized Excess Debt for T&T (USD million).

Year	Beginning Excess Debt	Fiscal Surplus	Principal	Interest	Ending Excess Debt
1	$2,072.57	$268.41	$164.78	$103.63	$1,907.79
2	$1,907.79	$268.41	$173.02	$95.39	$1,734.77
3	$1,734.77	$268.41	$181.67	$86.74	$1,553.10
4	$1,553.10	$268.41	$190.75	$77.66	$1,362.35
5	$1,362.35	$268.41	$200.29	$68.12	$1,162.06
6	$1,162.06	$268.41	$210.30	$58.10	$951.76
7	$951.76	$268.41	$220.82	$47.59	$730.94
8	$730.94	$268.41	$231.86	$36.55	$499.08
9	$499.08	$268.41	$243.45	$24.95	$255.63
10	$255.63	$268.41	$255.63	$12.78	$0.00

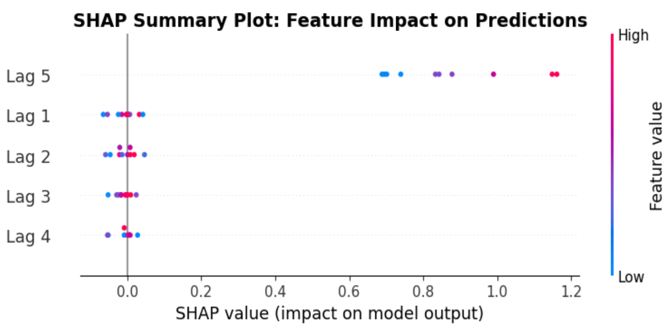

Figure 7. SHAP Results.

- MAE: 641.75
- RMSE: 834.28
- MAPE: 2.75%

These predictive accuracy diagnostics indicate that the mHC model achieves a reasonable level of predictive accuracy while still leaving room for improvement in magnitude tracking. Therefore, a MAE of 641.75 reflects the average absolute deviation between predicted and actual GDP values. The RSME of 834.28 implies that on average, the model's predictions deviate from actual GDP by US$834.28 million. While US$834.28 million is a large number, the GDP for T&T in 2024 was US$22,796 million. The MAPE was the better measure of the relative predictive accuracy. The MAPE of 2.75% indicates that the model's forecasts are, on average within roughly 2.75% of actual GDP values, a level which is acceptable for macroeconomic time-series forecasting. Taken together, these diagnostics suggest that the mHC model captures the broad nonlinear dynamics of GDP reasonably well.

The results of the cross validation, shown in Figure 6, demonstrate that the model achieves generalization with minimal overfitting, as evidenced by the stable metrics across all three folds.

Regarding the scaling of the metrics presented in Figure 6, the MAE have been transformed to the original scale. However, the R^2 values are inherently scale-invariant, requiring no transformation.

The SHAP values analysis displayed in Figure 7 reveals that the mHC model has successfully identified a meaningful and interpretable feature importance hierarchy, wherein Lag 5 emerges as the dominant predictor followed by Lag 1, indicating that the model appropriately weights both medium-term cyclical patterns and recent momentum in GDP dynamics.

4.3. Amortization of Excess Debt

T&T's total public debt was 75.6% of GDP in 2024. With a 2024 GDP of US$22,796 million, this corresponds to a debt level of US$17,233.88 million. The forecasted GDP for 2034 is US$25,268.8501 million.

US$25,424.21 million. At that time, a public debt of 60% of GDP would amount to US$15,161.310 million. This is US$2,072.57 million less than T&T's 2024 debt.

Table 4. ARIMA(1,1,0) Forecast of T&T's GDP.

Step/ Year	Forecast
Step 1	23017.39
Step 2	23105.11
Step 3	23139.89
Step 4	23153.68
Step 5	23159.15
Step 6	23161.32
Step 7	23162.18
Step 8	23162.52
Step 9	23162.66
Step 10	23162.71

In 2024, the Government of the Republic of Trinidad and Tobago (GORTT) recorded a total public expenditure of TT$57,875 million (approximately US$8,903.85 million), and total public revenue of TT$48,313.40 million (approximately US$7,432.83 million). This resulted in a fiscal deficit of US$1,471.02 million.

The mHC can be used to forecast T&T's total public revenue and total public expenditure up to 2034, enabling the calculation of the annual fiscal deficit and the corresponding increase in public debt.

As can be seen in Table 2, if T&T continues to incur fiscal deficits each year, it could experience in rise in debt by US$25,181.27846 million by 2034. If this is added to T&T's existing debt, this is raises T&T's debt to US$42,415.16 million or 167.85% of GDP by 2034. There is a need to avoid such a situation. This can be done through fiscal policy.

The target is for the GORTT to reduce its public debt from US$17,233.88 million in 2024 to US$15,161.310 million by 2034; a reduction of US$2,072.57 million; to achieve a debt-to-GDP ratio of 60%. To ensure this debt path is fiscally sustainable, the GORTT must generate sufficient primary fiscal surpluses over the next decade to service and amortize the excess debt.

Because fiscal adjustments occur over time, and money loses value over time, it is appropriate the model the required annual primary surplus as a level annuity that, when discounted at an interest rate (here assumed to be 5% per annum), has a present value equal to the debt reduction target of US$2,072.57 million.[1] This is precisely what the Present Value Interest Factor of an Annuity (PVIFA) framework provides.

$$Required\ Annual\ Surplus = \frac{Debt\ Reduction\ Target}{PVIFA(r,n)} = \frac{1{,}979.35}{\left[\frac{1-(1+r)^{-n}}{r}\right]} \quad (17)$$

where r=0.05 and n=10.

[1] The discount rate of 5% is justifiable it represents a moderate expected rate of return and opportunity cost of capital for investments with a perceived low to moderate level of risk.

Discounting future surpluses at the 5% rate is reflective of the opportunity cost of capital or borrowing for T&T. Without discounting, policymakers can underestimate the magnitude of near-term adjustments, leading to insufficient consolidation. By applying the PVIFA formula, the analysis translates a multi-year debt reduction goal into an annual fiscal target, which facilitates the alignment of short-term fiscal adjustments with long-term sustainability in a manner consistent with the IMF and the World Bank debt sustainability analysis practices.

The proposed approach is firmly rooted in the Intertemporal Government Budget Constraint, a cornerstone of debt sustainability analysis. The IGBC states that for public debt to be sustainable, the present value of future primary surpluses must equal or exceed the current stock of debt net of any sustainable baseline level. In T&T's case, the excess debt of US$2,072.57 million is above the 60% GDP threshold. Therefore, it is treated as an obligation that must be offset by fiscal discipline. By modeling the required annual primary surplus as a constant annuity, the framework ensures that the cumulative discounted value of these surpluses exactly covers the present value of the debt reduction target. This satisfies the IBC condition and guarantees that the government's fiscal path is consistent with long-run solvency, assuming that the forecasted GDP, and macroeconomic conditions hold and there is no new debt accumulation. The amortized excess debt is displayed in Table 3.

As can be seen from Table 3, the required annual fiscal surplus needed to amortize the excess debt of US$2,072.57 million over a 10-year horizon was calculated to be US$268.41 million. This value is the direct implication of the IGBC; it ensures that the stream of future primary balances is sufficient, in present-value terms, to eliminate the debt stock above the sustainable threshold.

4.4. ARIMA model

For comparability, the ARIMA model is specified. The Box-Jenkins methodology was applied through its iterative cycle of identification, estimation, and diagnostic checking. In the identification phase, the stationarity of the GDP series was assessed using the Augmented Dickey-Fuller test, which indicated non-stationarity, necessitating first-order differencing (d=1). The autocorrelation function (ACF) and partial autocorrelation function (PACF) plots of the differenced series were then examined to identify potential orders for the autoregressive (AR) and moving average (MA) components. In the estimation phase, multiple ARIMA models ranging from (1,1,0) to (3,1,3) were estimated. Finally, in the diagnostic checking phase, the models were evaluated based on the Akaike Information Criterion (AIC), with the ARIMA(1,1,0) model yielding the smallest AIC value, indicating the best relative fit among the candidate specifications. The ARIMA (1,1,0) model produced the linear fore

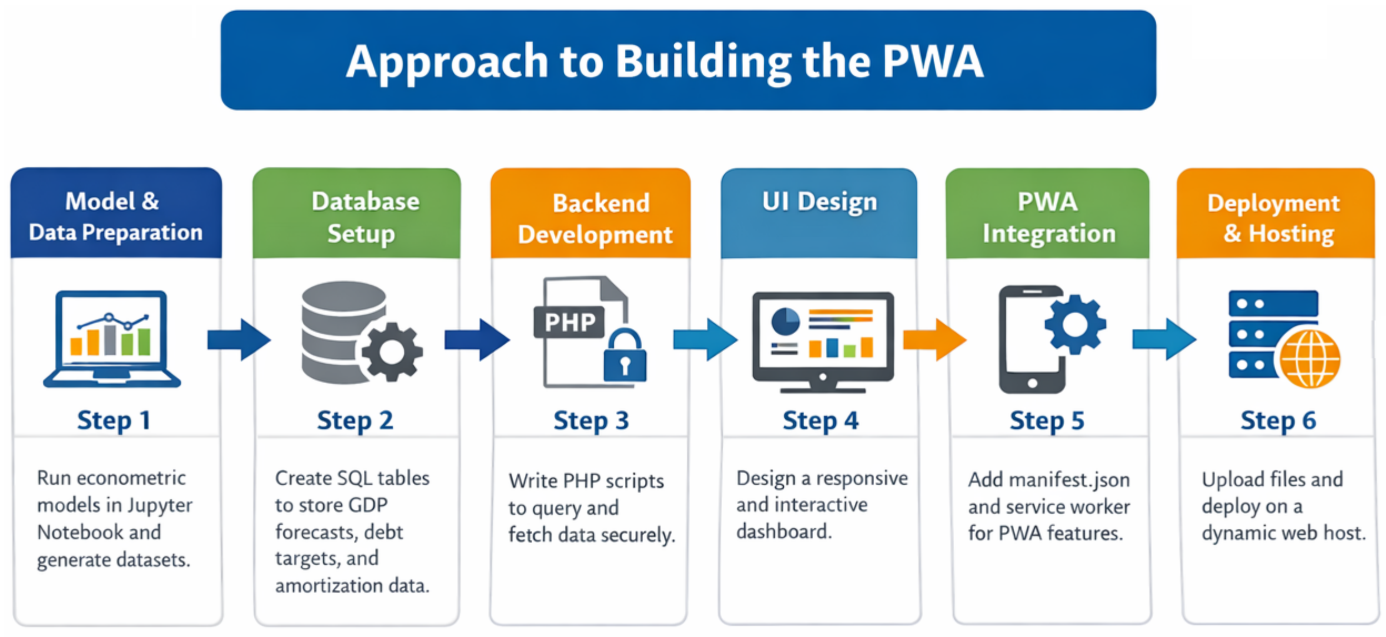

Figure 8. Approach to Building the PWA.

Table 5. Wilcoxon Signed-Rank Test on Multiple Metrics Results.

Metric	mHC Mean	ARIMA	P-value
Non-linearity	663.72	23.34	0.0020
Step Changes	1076.48	16.15	0.0039
Bootstrap Means	25261.95	23138.02	<0.0001
Volatility	892.13	49.16	0.0078

casts seen in Table 4, the converge to approximately US$23,162.71 million by 2034:

- MAE: 713.06
- RMSE: 933.85
- MAPE: 3.92%

While the model demonstrated strong in-sample fit with a MAPE of 3.92%, the fundamental assumptions of the underlying ARIMA methodology present significant concerns for this application. ARIMA models assume linearity in the underlying data generating process and normally distributed errors. However, preliminary diagnostic tests reveal that the GDP data exhibits non-linear characteristics and departs from the normal distribution, violating these core assumptions.

T&T's total public debt was 75.6% of GDP in 2024. With a 2024 GDP of US$22,796 million, this corresponds to a debt level of US$17,233.88 million. Using the ARIMA (1,1,0) model, the forecasted GDP for 2034 is US$23,162.71 million. At that time, a public debt of 60% of GDP would amount to US$13,897.626 million. This is US$3,336.254 million less than T&T's 2024 debt. This is a larger amount than the mHC forecast. This has a significant implication, as more debt has to be amortized and a larger fiscal surplus would be required.

Using the ARIMA(1,1,0) forecast, the target debt stock at 60% of 2034 is US$13,897.626 million. With current debt at US$17,233.88 million, the excess debt requiring amortization is US$3,336.254 million, which is substantially larger than the US$$2,072.57 million excess debt calculated using the mHC forecast. Applying the Present Value Interest Factor Annuity ($PVIFA_{r,n}$) at 5% discount rate over 10 years ($PVIFA_{r,n}$ = 7.7217), the required constant annual fiscal surplus is US$432.062 million, compared to only US$268.41 million under the mHC projection.[2]

Obviously, a larger fiscal surplus required would mean more austerity measures to be implemented by a government, which has significant implications for a country. This highlights the importance of using the appropriate forecasting model.

4.5. Comparison of the mHC vs the ARIMA model

The comparative evaluation of the models, as detailed in Table 5, provides evidence of the mHC model's different performance from the traditional ARIMA benchmark. The Wilcoxon Signed-Rank tests reveal that the mHC model is able to capture the underlying non-linearity (663.72 vs 23.34, p = 0.0020), and step changes (1076.48 vs 16.15, p = 0.0039) in the GDP time series. The bootstrap and volatility estimates also confirm that the mHC more reliably replicates the data-generating process's stochastic properties.

The next section discusses the building of the PWA.

[2] $PVIFA_{r,n} = \frac{1-(1+r)^{-n}}{r}$

where r is the discount rate; and n is the time period.

5. Discussion on building the PWA

The PWA will be built via several steps. Figure 8 displays the overall approach. As seen in Figure 8, the first part will involve creating the backend. The PWA would rely on the econometric models computed in Jupyter notebook. The backend's role is to store these results securely and make them available to the frontend via structure queries.

5.1. Model Execution and Data Generation

The computational analysis is performed offline using Python in Jupyter notebook, as web browsers cannot natively execute Python. The process follows the four-step analysis.

- Step 1 – GDP Forecast. The mHC model generates the 10-year out-of-sample forecast for T&T's real GDP.
- Step 2 – Target Debt Calculation. The target debt stock for year 10 (2034) is computed as 60% of the forecasted GDP for that year.
- Step 3 – Identifying the Target Debt or Excess Debt. The difference between the 2024 debt and the 2034 debt is the "target debt" (US$2,072.57 million) to be amortized.
- Step 4 – Amortization Schedule. Using a PVIFA formula (consistent with the IMF's debt sustainability frameworks), the constant annual fiscal surplus required to repay the targeted debt over 10 years is calculated.

The results of this model are the datasets that will populate the application's database.

5.2. Database Schema Creation with SQL

A local development stack such as XAMPP (which includes MySQL and PHP) can be used to create the database. SQL commands can be used to establish the necessary structure.

First, a database should be created. For instance, database called *fiscal_sustainability_tt* can be created. Subsequently, individual tables are created with SQL commands for each component of the analysis. A table can be created for the GDP forecast and populated with the forecasted values using SQL INSERT commands. Similarly, a table for the debt target can be created and updated with the relevant information. Next, create the table for the fiscal forecast. This is followed by creating a table containing the amortization schedule.

5.3. Backend Logic with PHP

Once all the SQL tables are created, the next step involves creating the PHP files with the server-side logic. The PHP files are processed on the server and act as the intermediary between the SQL database and the users' browser. Importantly, PHP files with the *.php* extension are used rather than .html files to facilitate security. Sensitive data like the database credentials and the logic of SQL queries reside exclusively on the server within PHP files, inaccessible to users through their browsers. If this logic were embedded in .html files, it would be exposed, creating significant database vulnerabilities.

PHP can execute the SELECT SQL queries to fetch the data from the database. A similar structure can be used for each data component: such as *gdp_forecast.php* and *fiscal_forecast.php* files. The PHP files can query and display the data from their respective tables.

5.4. User Interface Design

The user interface should be clean, responsive, and intuitive. The *index.php* can serve as the landing page, featuring buttons that can direct users to a dashboard displaying key information.

The dashboard can display the key metrics, namely the 2024 debt, the 2034 debt target, the excess debt to be amortized, and the annual surplus required. This can be followed by the interactive amortization table.

There can complementary figures. For instance, JavaScript can be used to show a chart within a PHP file to display the 10-year mHC GDP forecast. Similarly, JavaScript can be used to display charts of the forecasted total public revenue, total public expenditure, and the resulting fiscal deficits, highlighting the gap that needs to be closed by the calculated surplus.

Furthermore, CSS media queries can implement a responsive design, ensuring that the tables and charts render correctly across different user devices.

5.5. Making the Platform into a PWA

Transforming the web platform into a PWA requires two key files, namely the web app manifest, and a service worker. The web app manifest is a JSON file that defines how the app appears to the user and how it can be launched. The *manifest.json* file specifies the application name, icons, theme colors, and display preferences. The manifest should be linked in the <head> of the *index.php* file.

The service worker is a JavaScript file responsible for caching, enabling offline functionality and fast loading. The service worker caches the core application shell on the user's first visit. On subsequent visits or if the network is lost, the service worker serves the cached shell, allowing the app to load and display the most recently fetched data.

The service worker is registered in the main *index.php* file, and other core PHP files through a <script> tag, typically just before the closing </body> tag.

5.6. Deployment and Hosting

Dynamic websites and dynamic PWAs cannot be hosted on static hosting services (like GitHub Pages or Netlify) as they rely on server-side processing and a live database. Therefore, the PWA should be hosted on a dynamic web hosting provider that supports the LAMP (Linux, Apache, MySQL, PHP/Perl/Python) or LEMP (Linux, eNgine-X, MySQL/MariaDB, PHP/Perl/Python) stack. This hosting requirement typically incurs a fee for services. The developer of such a PWA would need to purchase an appropriate hosting package. Deployment involves uploading all *.php* files, *manifest.json*, and service-worker.js files on the server via the hosting panel's file manager. Using the hosting control panel, the developer can create a new MySQL database with the same names that were used in the local XAMPP environment. The names must match the names used in the PHP code, otherwise the application will fail to connect and function properly.

5.7. Limitations

The tool forecasts GDP to 2034 and computes a target debt level corresponding to 60% of that forecasted GDP. However, as with any forecast, the actual GDP in 2034 may be higher or lower than the projected value; no forecast can ever be 100% accurate. Consequently, the computed 60% debt-to-GDP threshold could also differ from the actual 60% debt in 2034. This uncertainty underscores the rationale for selecting the mHC model; it is specifically designed to capture complex, non-linear dynamics like macroeconomic time series such as GDP. Diagnostic evaluation of the model yielded a MAPE of 2.75%, which is very good in the context of macroeconomic forecasting. Currently, the computational workflow is conducted offline; Jupyter and Python are not executed live on the server. Ideally, all calculations should be performed server-side; requiring a server environment with Jupyter and Python installed, capable of running the mHC model, writing the results to SQL database tables, and the allowing PHP to query those tables to display the output in the PWA.

The present framework allows the PWA to only perform debt sustainability analysis for countries which the model has already been run offline and the resulting data have been loaded into the SQL tables. While this may appear to be a limitation, it actually presents an opportunity. Once key fiscal data; such as GDP, total public debt, total public revenue, and total public expenditure; are available for a country, the analysis can be executed offline, and the SQL tables can be updated accordingly. This approach enables the PWA to be incrementally extended to additional countries as data becomes available.

The next section highlights the contributions of this study.

6. Contributions

The contributions of this study are as follows.

- First, this study pioneers the empirical use of the Manifold-Constrained Hyper-Connection model; a novel deep learning architecture recently introduced [32]; for univariate forecasting of real GDP in a small open economy. While the mHC model was theoretically proposed [32], this work represents one of the first practical applications in macroeconomic forecasting, especially within a debt sustainability context. Through using the mHC's ability to capture complex nonlinear dynamics and structural constraints in time-series data, the study generates a 10-year out-of-sample GDP forecast for T&T.
- Second, is a methodological contribution through the design of a pragmatic debt sustainability framework tailored for countries that cannot apply the IMF's official DSA tolls, namely the MAC DSA or the LIC DSA, due to data limitations. Rather than treating debt sustainability as inaccessible without vast datasets, this study shows how core principles of fiscal solvency can be operationalized with minimal yet sufficient data: current debt stock, a GDP forecast, and a policy-relevant debt threshold.
- The study explicitly anchors its analysis in the Intertemporal Government Budget Constraint, ensuring theoretical rigor. It translates this abstract macroeconomic principle into a concrete, actionable fiscal target: a constant annual fiscal surplus sufficient to amortize excess debt over a decade. Using the PVIFA at a 5% discount rate, the study calculates that a steady annual fiscal surplus of US$268.41 million is required to reduce the excess debt (US$2,072.57 million) and achieve a 60 % debt-to-GDP ratio by 2034.
- Fourth, by combining a cutting-edge machine learning technique (mHC) with classical public finance theory (IGBC), and integrating it into a PWA, this study is at the intersection of econometrics, data science, fiscal policy, and computer science. The PWA offers several advantages, including cross-platform compatibility and offline functionality, which ensures that analysis can be conducted on a wide range of devices and in environments with unreliable internet connectivity, a common challenge in many developing regions. Furthermore, by packaging the econometric methodology within a PWA, it reduces the technical expertise required to conduct rigorous debt sustainability analysis.
- Fifth, the application of this framework to T&T provides an empirical contribution, which is particularly useful given the absence of a debt sustainability analysis specifically tailored to the country. The selection of T&T as a case study is also strategically important as it embodies many of the structural characteristics that make debt

sustainability challenging for small open economies, namely high dependence on a narrow export base, exposure to commodity price volatility, vulnerability to external macroeconomic shocks, and limited fiscal space for countercyclical policy responses.

The next section concludes this study.

7. Conclusion

The main objective of this study was to propose and design a computational framework for a PWA to derive the fiscal surplus required to reduce a country's the debt-to-GDP ratio to a sustainable level in 10 years. Using T&T as the empirical case study, the research addressed the policy-relevant question of what constant annual fiscal surplus would be necessary needed to lower the country's debt-to-GDP ratio from 75.6% to the benchmark level of 60% in a decade. The proposed framework integrates mathematical modeling with public finance theory to ensure that fiscal targets are grounded in both quantitative rigor and macroeconomic debt sustainability principles.

For forecasting purposes, the methodology employs the novel Manifold-Constrained Hyper-Connection model, which operates by restricting economic representations to a learned, low-dimensional manifold. This approach, inspired by differential geometry, ensures that GDP projections evolve along a structure state space, enhancing the stability and interpretability of the long-term outlook. The forecasted GDP is then fed into a PVIFA calculation to convert the debt reduction target into an annual fiscal surplus target. The mHC model produced a 10-step ahead out-of-sample forecast for T&T's GDP, culminating in a 2034 estimate of US$25,268.85 million. The associated MAPE for these forecasts was 2.75%, indicating a good level of predictive accuracy suitable for informing medium-term fiscal policy. Based on this forecast, and given T&T's 2024 debt of US$17,233.88 million alongside a 2034 target debt level of US$15,161.310 million (equivalent to 60% of projected GDP), the required total debt reduction amounts to US$2,072.57 million. The amortization calculation, assuming a 5% discount rate, results in a constant annual fiscal surplus of US$268.41 million over 10 years. This answers the first part of the research question.

After the model is run in Jupyter and results are generated, they are fed into a SQL database. The database stores the 10-year GDP forecasts, the calculated debt reduction target, the required annual surplus, and the fiscal amortization schedule. Server-side logic, written in PHP, securely queries this database and dynamically generates the content displayed to the user, keeping sensitive credentials and calculation logic protected in the server. The user interface is constructed to be clean and responsive, featuring a dashboard that presents key fiscal metrics, interactive tables, and JavaScript-powered charts for visualizing the GDP forecast and fiscal path, all styled with CSS responsive design for cross-device compatibility.

8. Declarations

8.1. Author Contributions

Don Charles: Conceptualization, Methodology, Software, Validation, Formal analysis, Investigation, Data Curation, Writing, Editing.

8.2. Institutional Review Board Statement

Not applicable.

8.3. Informed Consent Statement

Not applicable.

8.4. Data Availability Statement

T&T's real GDP (GDP 2018 constant prices) was obtained from CEPALSTAT at https://statistics.cepal.org/portal/cepalstat/dashboard.html?theme=2&lang=en.

T&T's debt to GDP ratio, the GORTT total public revenue and T&T total public expenditure were obtained from the CBTT online database at https://www.central-bank.org.tt/statistics/data-center-trial/.

The codes for the mHC model and the ARIMA model forecast, the pre-tests, and the diagnostic tests are available at https://github.com/doncharles005/mHCv2.

8.5. Acknowledgment

Not applicable.

8.6. Conflicts of Interest

The authors declare no conflicts of interest.

9. References

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