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Review

Exploring Quantum Machine Learning in Solving Complex Optimization Problems: Algorithms and Insights

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Abstract: Optimization problems across domains such as logistics, finance, and artificial intelligence often involve complex and NP-hard formulations that are computationally challenging for classical algorithms due to scalability and efficiency limitations. The study aims to systematically investigate the role of Quantum Machine Learning (QML) in addressing complex optimization problems and to analyze its advantages over traditional optimization techniques. A comprehensive survey and comparative analysis of key QML algorithms, including Quantum Approximate Optimization Algorithm (QAOA), Variational Quantum Eigensolver (VQE), Quantum Neural Networks (QNNs), and Quantum Support Vector Machines (QSVMs), is conducted by examining their working principles, optimization capabilities, and real-world applications. The findings indicate that QML algorithms demonstrate significant potential in exploring large solutions spaces efficiently, achieving faster convergence, and providing improved optimization performance compared to classical approaches, although challenges such as quantum noise, scalability, and hardware limitations remain. The novelty of this study lies in providing a unified and critical comparative framework that integrates multiple QML optimization algorithms, highlights their practical feasibility, and identifies key research gaps hindering their real-world deployment. The implications of this research provide valuable insights for researchers and practitioners in selecting appropriate QML techniques and emphasize the need for advancements in hybrid quantum -classical systems, algorithms design, and quantum hardware to enable practical large-scale optimization.

Keywords: Quantum Machine Learning; Optimization algorithms; Quantum Approximate Optimization Algorithm; Variational Quantum Eigensolver; Quantum Neural Networks; Real-world challenges.

1. Introduction

Optimization challenges are vital across several sectors, such as logistics, banking, and artificial intelligence (AI) (Doumpos et al. [1] and Adeniran et al. [2]), where identifying the optimal solution from an extensive array of options is crucial. Logistics optimization difficulties include minimizing transportation expenses, enhancing delivery efficiency, and controlling complex supply networks (Baloch et al. [3] and Caunhye et al. [4]). Classical algorithms frequently face challenges in scaling with the dynamic and extensive datasets associated with these issues (Xu et al. [5]). In finance, portfolio optimization involves handling risk and return. It requires the resolution of multi-objective optimization problems within fluctuating market circumstances and regulatory limitations (Bradshaw et al. [6] and Mogbojuri et al. [7]). These issues rapidly grow unmanageable as the complexity of the financial environment escalates. Artificial intelligence also encounters optimization problems in jobs like training deep learning models (Aggarwal et al. [8]), where identifying the ideal parameters to minimize loss functions is computationally intensive. As datasets increase in size and complexity, traditional optimization approaches, including gradient-based algorithms (Wang et al. [9]) and evolutionary techniques, are inadequate, particularly for NP-hard issues.

Quantum Machine Learning (QML) is an emerging field that combines the computational capabilities of quantum computing with machine learning methods to improve their efficiency and scalability (Schuld et al. [10] and Ciliberto et al. [11]). Quantum Machine Learning (QML) offers a viable answer in this context. Using quantum concepts like superposition (Shukla et al. [12]) and entanglement (Ge et al. [13]), QML algorithms may parallelly investigate extensive search areas (Dai et al. [14]), which presents possible accelerations for these complex optimization challenges. In contrast to conventional approaches that examine solutions sequentially, QML may assess several states concurrently (Yang et al. [15]), possibly alleviating computing bottlenecks and facilitating expedited convergence to optimal solutions.

Despite the rapid development of Quantum Machine Learning (QML) for optimization, several research gaps remain. Existing studies primarily focus on individual QML algorithms, such as QAOA, VQE, QSVM, or QNN, without providing a comprehensive comparative analysis of their optimization capabilities, practical feasibility, and limitations within a unified framework. Furthermore, limited attention has been given to systematically identifying the challenges that hinder real-world deployment, including scalability constraints, quantum hardware limitations, and algorithmic performance trade-offs. To address these gaps, this study presents a comprehensive and structured comparative analysis of major QML-based optimization algorithms. The novelty of this work lies in integrating multiple QML optimization approaches into a single unified analytical framework, critically evaluating their optimization performance, application suitability, and existing limitations.

Unlike previous survey studies that discuss algorithms independently, this research provides a focused comparison specifically from an optimization perspective. The main purpose of this research is analyzed and evaluate the effectiveness of Quantum Machine Learning algorithms in solving complex optimization problems, identify current research challenges, and highlight future research directions. This study aims to support researchers in understanding the strengths and limitations of QML-based optimization and facilitate the development of more efficient and scalable quantum optimization solutions.

The rest of the paper is structured as follows; Section 2 presents research methodology. Section 3 explores existing research. Section 4 delves into various quantum approaches. Section 5 discusses key challenges. Use cases and real-world applications are examined in Section 6. Section 7 summarizes key findings and Section 8 outlines potential research directions.

2. Research Methodology

DL-based solutions have been used in many fields, i.e., disease detection in humans (Raza et al. [16], Saeed et al. [17], Khan et al. [18], Saeed at al. [19], Khan et al. [20], Naqvi et al. [21], Saeed at al. [22], and Nawaz et al. [23]), plants (Saeed et al. [24], and Saeed et al. [25]), and various multidisciplinary fields (Saeed et al. [26], Ishtiaq et al. [27], Raza et al. [28], Saeed at el. [29], Nawaz et al. [30], and Nawaz et al. [31]).This study adopts a systematic literature review methodology to analyse the role of Quantum Machine Learning in solving complex optimization problems. The research methodology consists of two main phrases: data collection and data analysis.

2.1. Data Collection Method

The data for this study were collected from reputable scientific databases, including IEEE Xplore, ScienceDirect, SpingerLink, Wiley Online Library, and Google Scholar. Relevant research articles published between 2015 and 2024 were considered. The following keywords were used during the during the search process “Quantum Machine Learning”, “Quantum Optimization”, “Quantum Approximate Optimization Algorithm (QAOA)”, “Variational Quantum Eigensolver (VQE)”, “Quantum Support Vector Machine (QSVM)”, and “Quantum Neural Networks (QNN)”.

2.2. Data Analysis Method

The collected data were analyzed using qualitative comparative analysis. Each selected study was examined to evaluate the optimization performance, application domain, advantages, and limitations of the proposed QML algorithms. The algorithms were compared based on their efficiency, scalability, convergence capability, and practical feasibility. Furthermore, the analysis focused on identifying gaps, challenges, and future research directions. The comparative evaluation enabled a structured understanding of how different QML algorithms contribute to solving complex optimization problems and their potential advantages over classical approaches.

Figure 1 illustrates the systematic literature review process used in this study. Initially research article was identified from major scientific databases including IEEE Xplore, Spinger, ScienceDirect, and Google Scholar. In the identification phase, relevant articles were collected using predefined keywords. In the screening phase, duplicate and irrelevant articles were removed based on title and abstract review. In the eligibility phase, full-text articles were evaluated based on inclusion and exclusion criteria. Finally, the most relevant studies were selected for de-

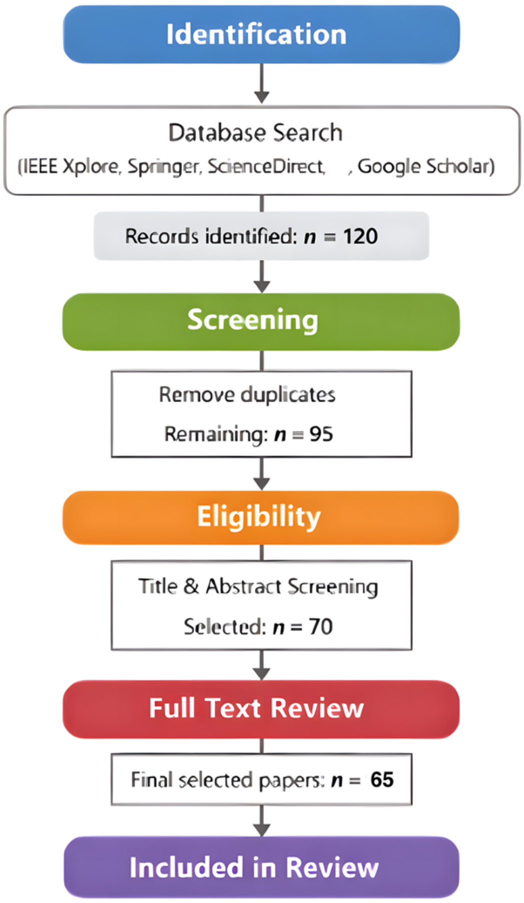

Figure 1. Systematic Literature review process flowchart.

tailed analysis. This systematic process ensured transparency and reliability of the review methodology.

3. Related work

Classical optimization methods are essential for identifying optimum solutions across many domains by reducing or maximizing an objective function under certain restrictions. Methods such as linear programming, genetic algorithms, and simulated annealing are frequently used to resolve optimization issues effectively. Many researchers have presented their work as follows:

Sejati et al. [32] used linear programming to enhance irrigation water management to maximize agricultural production and profit. Their research utilized the POM QM program to develop three alternative farming patterns using corn and peanuts. The cropping pattern for MTI demonstrated the best profitability. The proposed method provided an annual profit of Rp 11,566,000,000. Mesquita-Cunha et al. [33] addressed multi-objective optimization problems with generation methods demonstrating the Pareto front. The authors developed three techniques to produce representative sets of the Pareto front, focusing on cardinality, coverage, and uniformity. Using the ε-constraint method, the algorithms were evaluated on binary and integer linear programming problems. The cardinality method is flexible, optimizing uniformity and coverage per the representation size. Khoulenjani et al. [34] investigated using optimization methods in feasibility analyses for building projects under uncertainty. Their system incorporated uncertainty analysis and optimization. A numerical example showed that optimization approaches enabled more robust evaluations by incorporating diverse uncertainty scenarios and enhanced the dependability of feasibility studies in the construction sector.

Ahmad et al. [35] investigated optimizing operating parameters to maximize hydrogen production from wastewater via electrolysis. The study utilized a Box-Behnken design (BBD) with an L17 array, which incorporates Response Surface Methodology (RSM) in conjunction with Genetic Algorithms (GA) and Particle Swarm Optimization (PSO) to create predictive models and optimize essential parameters such as catalyst amount, electrode voltage, and electrolysis time. The results showed that RSM exhibited superior performance compared to GA and PSO in optimizing parameters. The research showed the efficacy of integrating RSM, GA, and PSO for optimizing wastewater electrolysis. Ghali et al. [36] focused on optimizing tolerance allocation in design and manufacturing by developing a method utilizing the genetic algorithm (GA). The proposed method minimizes total cost while adhering to functional and manufacturing requirements by incorporating difficulty coefficients into the tolerance equations. Incorporating Failure Mode, Effects, and Criticality Analysis (FMECA) alongside the Ishikawa diagram into the Difficulty Coefficient Computation (DCC) established a thorough framework for the optimization process.

Babu et al. [37] examined the challenges related to data security, privacy, and energy efficiency within the Internet of Things (IoT) networks. An energy optimization and node deployment strategy were proposed, integrating Genetic Algorithm (GA) for energy optimization with Mixed Integer Linear Programming (MILP) for strategic node replacement. Furthermore, blockchain technology was incorporated to improve data privacy and access control. The proposed model demonstrated superior performance to current state-of-the-art models, specifically in network lifetime and throughput. It offers a dependable and efficient solution for IoT networks by optimizing residual energy and reducing the necessary node set for network servicing. Casarin et al. [38] introduced two stochastic optimization-based Simulated Annealing algorithms to tackle challenges associated with market risk measures such as Value-at-Risk and Expected Shortfall, especially in the context of integral constrained optimizations. The algorithms are structured to manage criterion functions that are not necessarily differentiable and involve complex integral limitations, which present challenges for analytical reduction.

Kumar et al. [39] presented a hybrid approach that combines genetic algorithms with simulated annealing to determine the optimal placement of electric vehicle charging stations, improving the distribution resilience of the network. The study aimed to optimize the positioning of charging stations to enhance the distribution efficiency and reliability of the network. The approach integrates the global search functionality of genetic algorithms with the local optimization efficiency of simulated annealing, resulting in a comprehensive solution for handling distribution network resilience challenges. Ominato et al. [40] presented an improved variant of the Grover Adaptive Search (GAS) algorithm, designed to address binary optimization challenges more efficiently by reducing the required number of queries. The study introduces a novel approach for determining the number of queries necessary to enhance the Grover Search (GS) target state, which functions as a subroutine within GAS. Simulation results for 40-bit problems showed that the proposed method achieves a quadratic speedup comparable to the original GAS while simultaneously decreasing the number of queries by 22%. Qiu et al. [41] introduced a distributed variant of Grover's algorithm, which minimizes the qubit requirements and provides a linear enhancement in time complexity. The distributed algorithm employs K computing nodes, resulting in a query complexity of $O(\sqrt{N}/K)$, which is an improvement over Grover's $O(\sqrt{N})$. This enhancement is particularly beneficial for large-scale unstructured search problems. The authors also developed an algorithm designed to construct quantum circuits for Boolean functions represented in conjunctive normal form (CNF), thereby improving the practicality of their approach.

Shukla et al. [42] improved Grover's search algorithm when the total number of search items (N) does not conform to a power of 2. The method presented introduces a technique for preparing uniform quantum superposition states across a subset of computational basis states. This approach reduces Oracle calls and Grover iterations by 29.33% in specific scenarios. The improvement represents a notable advancement compared to conventional methods, which address these situations by rounding N to the closest power of 2. The algorithm demonstrated a gate complexity and circuit depth of $O(\log_2(N))$ while not necessitating any ancilla qubits. Celik et al. [43] analyzed Grover's quantum search algorithm through classical simulations. The authors focused on calculating amplitudes and probabilities for locating a single marked state across different qubit states (n=5, 10, 15, 20, 25, 27). The results showed that $O(\ln(N))$ iterations can identify the marked state. This signifies a significant enhancement in locating a single marked element within an unsorted search space comprising N items. The summary of related work is shown in Table 1.

4. Quantum Machine Learning Algorithms for Optimization

4.1. Quantum approximate optimization algorithm (QAOA)

The Quantum Approximate Optimization Algorithm (QAOA) represents a hybrid approach that integrates quantum and classical computing techniques to address combinatorial optimization challenges. The operation involves converting a discrete optimization problem into a classical one that utilizes continuous circuit parameters. The QAOA structure consists of alternating quantum gates implementing the problem Hamiltonian and a mixer Hamiltonian on the qubits. This is succeeded by a classical optimization loop that fine-tunes the parameters controlling these gates. The algorithm operates iteratively to minimize or maximize the objective function. The effectiveness of QAOA is attributed to its capacity to sample high-quality solutions from a superposition of potential solutions.

QAOA has been extensively utilized in numerous optimization problems, including MaxCut, the traveling salesperson problem (TSP) (Hahsler et al. [44]), and portfolio optimization (Gunjan et al. [45] and Ban et al. [46]). The algorithm demonstrates significant adaptability in addressing NP-hard problems (Hidalgo-Herrero et al. [47]), particularly where traditional methods encounter difficulties related to the expansive nature of the search space. In sectors such as logistics and finance (Van Nieuwenhuizen et al. [48]), QAOA has shown its capability by enhancing the efficiency of identifying optimal solutions for resource allocation, network optimization, and task scheduling. The system's ability to function effectively with noisy intermediate-scale quantum (NISQ) devices positions it as a leading option for achieving near-term quantum advantage in addressing complex combinatorial problems.

In QAOA, the Hamiltonian problem represents the optimization problem, which corresponds to the objective function that needs to be minimized (or maximized). The primary procedures in QAOA consist of alternating between the problem Hamiltonian H_c and a mixer Hamiltonian H_M, with the parameters γ and β being optimized during each iteration. The cost Hamiltonian is defined for an objective function C(x) is shown in Equation 1.

$$\boldsymbol{H_c} = \sum_{\boldsymbol{z}} \boldsymbol{C(z)|z\rangle\langle z|} \tag{1}$$

In this context, z denotes a bitstring representing a possible solution, while C(z) signifies the cost related to that solution. The algorithm starts with an equal superposition of all potential states as shown in Equation 2.

$$|\boldsymbol{\psi_0}> = \frac{\boldsymbol{1}}{\sqrt{\boldsymbol{2^n}}} \sum_{\boldsymbol{z}} |\boldsymbol{z}\rangle \tag{2}$$

Table 1. Summary of related work.

Reference	Approach used	Improvement	Limitations
Sejati et al. [32]	Linear programming	Enhanced irrigation water management, maximizing agricultural profit	Limited to specific farming conditions
Mesquita-Cunha et al. [33]	Multi-objective optimization	Improved Pareto front representation with optimized cardinality, coverage, and uniformity	Requires complex computation for larger datasets
Khoulenjani et al. [34]	Optimization in feasibility analysis	More robust feasibility evaluations under uncertainty	High computational cost
Ahmad et al. [35]	RSM, GA, and PSO	Optimized hydrogen production parameters	Requires hybrid modeling for accuracy
Ghali et al. [36]	Genetic algorithm	Optimized tolerance allocation in manufacturing	Computational intensity and dependency on predefined coefficient
Babu et al. [37]	GA and MILP with blockchain	Enhanced IoT security and energy efficiency	Requires blockchain implementation overhead
Casarin et al. [38]	Stochastic simulated annealing	Improved market risk optimization	Complexity in handling integral constraints
Kumar et al. [39]	Hybrid GA-simulated annealing	Optimized EA charging station placement	Scalability concerns
Ominato et al. [40]	Improved Grover adaptive search	Quadratic speedup with 22% fewer queries	Limited to specific binary optimization problems
Qiu et al. [41]	Distributed Grover's algorithm	Reduced qubit requirements, linear time complexity improvement	Practical implementation challenges
Shukla et al. [42]	Grover's algorithm optimization	Reduced oracle calls and iterations by 29.33%	Limited scalability to higher-dimensional problems
Celik et al. [43]	Classical simulation of Grover's algorithm	Enhanced market state identification	Limited practical application for large datasets

The Quantum Approximate Optimization Algorithm (QAOA) subsequently implements a sequence of quantum gates controlled by the parameters γ and β associated with applying the problem and mixer Hamiltonians, as shown in Equation 3.

$$|\psi_p(\gamma,\beta)\rangle = e^{-i\beta_p H_M} e^{-i\gamma_p H_c} \dots e^{-i\beta_i H_M} e^{-i\gamma_i H_c} |\psi_0\rangle \quad (3)$$

The final state $\psi_p(\gamma,\beta)\rangle$ depends on these parameters, which are classically optimized to maximize the expectation value of the cost functions, as shown in Equation 4.

$$\langle H_c\rangle = \langle \psi_p(\gamma,\beta)|H_c|\psi_p(\gamma,\beta)\rangle \quad (4)$$

The goal of QAOA is to find the set of parameters. γ,β maximize the expectation value. Numerous studies have explored the Quantum Approximate Optimization Algorithm (QAOA) and highlighted various aspects of its development, application, and optimization techniques. Falla et al. [49] presented the concept of QAOA parameter transferability specifically in relation to MaxCut problem instances. The research showed the transferability of optimal QAOA parameters across various classes of MaxCut problems. This approach effectively decreased the iterations needed for parameter optimization and addressed the challenge of barren plateaus in variational optimization. Wybo et al. [50] analyzed the Quantum Approximate Optimization Algorithm (QAOA) performance on random regular graphs, explicitly focusing on the MaxCut and maximum independent set problems. The authors demonstrated that the approximation ratios of QAOA improved with graph regularity in the MaxCut problem. At the same time, the authors decreased the maximum independent set problem as a result of the overlap gap property. Amosy et al. [51] introduced a fully

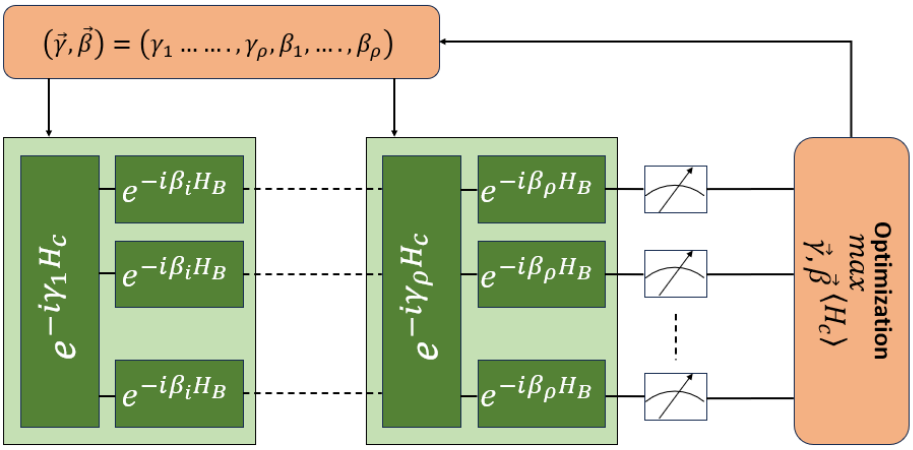

Figure 2. workflow of QAOA.

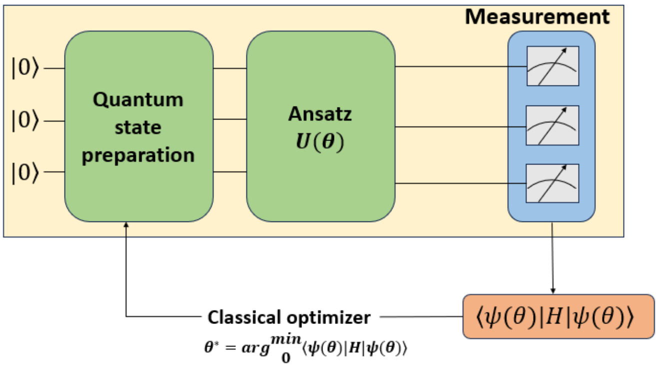

Figure 3. Workflow of VQE.

connected neural network to improve the Quantum Approximate Optimization Algorithm (QAOA) by predicting more effective initialization parameters. This advancement notably accelerates the convergence of QAOA when applied to the MaxCut problem on Erdős-Rényi graphs.

Ding et al. [52] implemented the Quantum Approximate Optimization Algorithm (QAOA) for molecular docking challenges in drug discovery. Their findings showed that the digitized-counter-diabatic QAOA exhibited superior performance to traditional QAOA, particularly regarding circuit depth and optimization efficiency. Santra et al. [53] established a connection between QAOA and quantum metrology by utilizing QAOA to generate highly squeezed states for solving combinatorial MaxCut problems, thereby enhancing the performance of quantum devices. Yanakiev et al. [54] introduced Dynamic-ADAPT-QAOA, a modified version of QAOA that aims to minimize circuit depth and enhance noise resilience, thereby improving its applicability for near-term quantum hardware. Liu et al. [55] presented a QAOA-based maximum likelihood detection solver designed for massive multiple-input and multiple-output (MIMO) systems. The findings demonstrated a quantum advantage in addressing detection challenges within communication systems. Cheng et al. [56] introduced double adaptive-region Bayesian optimization (DARBO) to enhance the performance of QAOA by accelerating the optimization process and mitigating quantum noise, as evidenced by superconducting quantum processors. Okada et al. [57] studied warm-start QAOA (WS-QAOA) and its efficacy in addressing MaxCut problems. The results showed that approximate solutions derived from QAOA improve fidelity and approximation ratios when implemented in shallow circuits. Ni et al. [58] introduced a Multilevel Leapfrogging Interpolation (MLI) strategy to minimize the computational costs associated with the parameter initialization of QAOA for deep circuits. The proposed approach achieves comparable quasi-optimal results to conventional methods.

Figure 2 shows the workflow of QAOA. The method starts with initializing parameters γ and β, which guide the evolution of the quantum circuit. The quantum state is explored by alternating between the Cost Hamiltonian (H_C) and the Mixer Hamiltonian (H_B) to encode problems and explore states. Measurements extract expectation values and input them into a traditional optimization loop to adjust γ and β iteratively. This method is repeated until convergence, which provides an estimated solution to the optimization issue.

4.2. Variational Quantum Eigensolver (VQE)

A hybrid quantum-classical approach called the Variational Quantum Eigensolver (VQE) is mainly used to address eigenvalue challenges, particularly in quantum chemistry and molecular optimization. Using quantum circuits, VQE minimizes the expectation value of the Hamiltonian, which reflects the system's overall energy, to estimate a molecular system's ground state energy. It achieves this by employing ansatzes, or parameterized quantum circuits, that are iteratively optimized with the help of a classical optimizer. When it comes to quantum chemistry difficulties, where understanding chemical characteristics and reactions requires knowing the ground state of a molecular system, VQE is very helpful. Researchers can simulate molecular interactions, predict reaction results, and optimize molecular geometries due to the algorithm's ability to efficiently tackle the electronic structure problem of molecules. VQE has the potential to model complicated molecules in molecular optimization that are now unmanageable by standard computers. It can be used, for instance, in material science to anticipate molecular conformations or in drug research to optimize molecules' binding energies. While VQE can deliver quantum benefits even on poor hardware, it is regarded as one of the most promising quantum algorithms for near-term quantum devices (NISQ era) despite the difficulties caused by noisy quantum hardware.

VQE minimizes the expectation value of a Hamiltonian, H, which represents the system's total energy. The goal is to find the ground-state energy E_0, which is the lowest eigenvalue of H. The VQE algorithm uses a parameterized quantum circuit $|\psi(\vec{\theta})\rangle$, where $(\vec{\theta})$ represents the parameters to be optimized. The energy expectation value is given in Equation 5.

$$\boldsymbol{E(\vec{\theta}) = \langle\psi(\vec{\theta})|H|\psi(\vec{\theta})\rangle} \quad (5)$$

The Hamiltonian H is typically decomposed into a sum of Pauli operators, as shown in Equation 6.

$$\boldsymbol{H = \sum_i c_i P_i} \quad (6)$$

where c_i are coefficients and P_i are Pauli operators. The quantum computer evaluates the expectation values. $\langle\psi(\vec{\theta})|P_i|\psi(\vec{\theta})\rangle$, while a classical optimizer adjusts the parameter $(\vec{\theta})$ to minimize $E(\vec{\theta})$. The optimization is performed iteratively until the energy converges to the ground-state energy, E_0.

VQE uses ansatzes such as the Unitary Coupled Cluster (UCC) ansatz, which is well-suited for simulating molecule electron correlations (Haider et al. [59]). The UCC ansatz expresses the wavefunction as an exponential of a cluster operator applied to a reference state. The parameters in the ansatz are then optimized to minimize the expectation value of the Hamiltonian. Matoušek et al. [60] improved the performance of VQE in chemical simulations by integrating it with the adiabatic connection (AC) approach. Strong electron correlation was captured using orbital-optimized VQE, and traditional AC corrections restored dynamical correlation effects. This method greatly enhanced the dissociation of N2 and the tetramethylene ethane biradical's electronic structure, demonstrating the algorithm's promise for quantum simulations on near-term quantum devices. Lim et al. [61] presented the Fragment Molecular Orbital/Variational Quantum Eigensolver (FMO/VQE) technique to address scaling challenges in quantum chemistry simulations. The authors achieved encouraging results for complicated molecules such as H_3O^+ and H_2O^+combining the fragment molecular orbital technique with VQE allows large chemical systems with fewer qubits to be efficiently simulated.

By adding the CAFQA technique, which makes use of conventional computing power to enhance the starting state for quantum simulations, Wang et al. [9] improved VQE. Their work showed the usefulness of VQE in actual quantum devices by showing faster convergence and improved energy values for molecules such as LiH and BeH_2 on a trapped-ion quantum computer. Tran et al. [62] presented variational denoising, an unsupervised learning method combined with VQE to reduce noise in quantum calculations. Their approach increased the accuracy and trainability of VQE on noisy quantum devices by improving energy estimations for molecular Hamiltonians, including H_2, LiH, and BeH_2.

Haider et al. [59] created the OpenVQE open-source program to simplify VQE calculations for quantum chemistry applications. The package's adaptive derivative techniques tools and compatibility with different quantum programming frameworks allow for efficient simulation of complex molecules with up to 24 qubits. Jiang et al. [63] used parametric Gaussian process regression (GPR) in an active learning framework to address the noise issues in VQE. Their method reduced the number of quantum processor evaluations by significantly increasing the accuracy of VQE outputs for the Heisenberg and Anderson impurity models. Wiśniewska et al. [64] demonstrated the VQE method's adaptability beyond quantum chemistry by adapting it to a data categorization task utilizing quantum circuits. The author addressed a credit sales categorization problem by combining VQE with the SWAP test, which provided a fresh method for applying quantum algorithms to decision-making.

Figure 3 shows the workflow of VQE. The method starts with quantum state preparation, which involves creating an initial state $|0\rangle$ on several qubits. A parameterized quantum circuit (Ansatz) U(θ) transforms the state based on adjustable parameters θ. After using the Ansatz, the quantum system is measured to get Hamiltonian expectation values. The data are supplied into a classical optimizer, which adjusts θ to minimize the energy expectation value, $\langle\psi(\theta)|H|\psi(\theta)\rangle$. This iterative procedure is repeated until convergence, which results in an estimate of the ground state and its accompanying energy. VQE is especially beneficial in quantum chemistry and materials research, where tackling eigenvalue issues traditionally involves computationally expensive.

4.3. Quantum Neural Network (QNNs)

Quantum Neural Networks (QNNs) (Abbas et al. [65]) are the quantum equivalents of conventional neural networks, designed to utilize the concepts of quantum physics for machine learning applications (Beer et al. [66]). Instead of using typical layers of neurons, QNNs use quantum circuits (Ezhov et al. [67]), where qubits are manipulated by quantum gates acting as operations. The benefit of QNNs is that they may take advantage of quantum phenomena such as entanglement and superposition, which can result in exponential speedups for learning tasks. These networks are designed to address challenging issues that traditional neural networks find difficult, especially in high-dimensional spaces, such as classification, pattern recognition, and data processing.

In domains like data categorization, image recognition, and quantum chemistry, where traditional approaches need many more resources to get the same results, QNNs are pretty helpful. Furthermore, QNNs are advantageous in quantum data processing and simulations (Tomas [68]) and have been suggested to solve optimization issues in quantum machine learning (QML). However, in near-term quantum technology, QNNs must also deal with noise sensitivity, gradient vanishing (barren plateaus), and training.

Similar to a conventional neural network, a quantum neural network (QNN) is often built using parameterized quantum gates applied in a layered manner. These quantum gates serve as the fundamental components of quantum circuits, which encode input data into qubit states and then carry out operations that generate a quantum state that represents the solution. For an input quantum state $|\psi_{input}\rangle$, the evolution of the state through a QNN can be described in Equation 7.

$$|\boldsymbol{\psi}_{\boldsymbol{input}}(\vec{\boldsymbol{\theta}})\rangle = \boldsymbol{U}_{\boldsymbol{L}}(\boldsymbol{\theta}_{\boldsymbol{L}}) \dots \dots \boldsymbol{U}_{\boldsymbol{1}}(\boldsymbol{\theta}_{\boldsymbol{1}})|\boldsymbol{\psi}_{\boldsymbol{input}}\rangle \quad (7)$$

Here, $U_i(\theta_i)$ represents a unitary operation parameterized by θ_i, the adjustable parameters of the network. The parameters $\vec{\theta} = \{\theta_1, \theta_2, \dots . \theta_L\}$ are optimized during training using quantum optimization algorithms. The cost function of a QNN, $C(\vec{\theta})$, can be defined as the expectation value of an observable O over the output quantum state, as shown in Equation 8.

$$\boldsymbol{C}(\vec{\boldsymbol{\theta}}) = \langle \boldsymbol{\psi}_{\boldsymbol{output}}(\vec{\boldsymbol{\theta}}) | \boldsymbol{O} | \boldsymbol{\psi}_{\boldsymbol{output}}(\vec{\boldsymbol{\theta}}) \rangle \quad (8)$$

The objective is to minimize or maximize the cost function by adjusting the parameters $\vec{\theta}$, often through gradient-based methods like Quantum Natural Gradient Descent or hybrid quantum-classical algorithms such as Quantum Approximate Optimization Algorithm (QAOA).

A no-go theorem was created by Zhang et al. [69] for employing QNNs to learn unknown quantum states. The authors showed that the likelihood of avoiding local minima reduces exponentially with the qubit count as the loss value falls below a crucial threshold and imposes fundamental restrictions on the scalability of QNNs. Additionally, the study looked at how quantum Fisher information affects the output state's sensitivity to QNN parameters. The problem-dependent power of quantum neural classifiers (QCs) for multiclass classification problems was investigated by Du et al. [70]. Their results showed that the generalization ability was only a corollary of the training loss in determining the power of QCs. According to the study, QCs have a U-shaped risk curve instead of the double-descent risk curve in traditional deep neural networks.

Remaining quantum neural networks, or ResQNets, were proposed by Kashif et al. [71] to address the barren plateau challenge in QNNs. Compared to traditional QNN designs, their work demonstrated enhanced training performance by integrating residual connections between quantum nodes. This method provides a viable way to improve QNNs' trainability and scalability. Das et al. [72] integrated a parameterized quantum circuit as an input layer and used quantum optimization methods to create a variational quantum neural network (VQNN) model. When their model was used for crack image classification and MNIST digit identification, it outperformed traditional QNNs regarding training accuracy and time complexity. Zhang et al. [73] presented a novel viewpoint on the trainability of QNNs. The research demonstrated that the loss function's variation range disappears rapidly with the number of qubits, placing severe limitations on gradient-based and gradient-free optimization techniques for QNNs.

In their assessment of high-level programming techniques for QNN creation, Markidis [74] concentrated on parameterized quantum circuits, quantum annealers, and the accompanying optimizers. The investigation shed light on QNN framework architecture and machine learning applications. Kaseb et al. [75] showed better generalization ability and robustness when using hybrid quantum-classical neural networks for power flow analysis compared to both quantum and classical neural networks. Their hybrid approach proved quite successful for deep learning-based power flow analysis, outperforming traditional techniques.

A Quantum Neural Network (QNN) uses quantum circuits to analyze and learn from input similarly to conventional neural networks but with the benefits of quantum computing. Figure 4 shows a typical QNN workflow in which an input quantum state $|\psi\rangle$ is prepared and passed via a multilayer quantum circuit with parameterized unitary operations U(θ). These layers perform entangling and rotating operations, allowing the network to encode and manipulate quantum data. The prediction or classification result is determined by measuring a specific qubit Y_{n+1}. The circuit parameters are iteratively tuned using classical gradient-based approaches to minimize a cost function, allowing QNNs to learn from both quantum and conventional input.

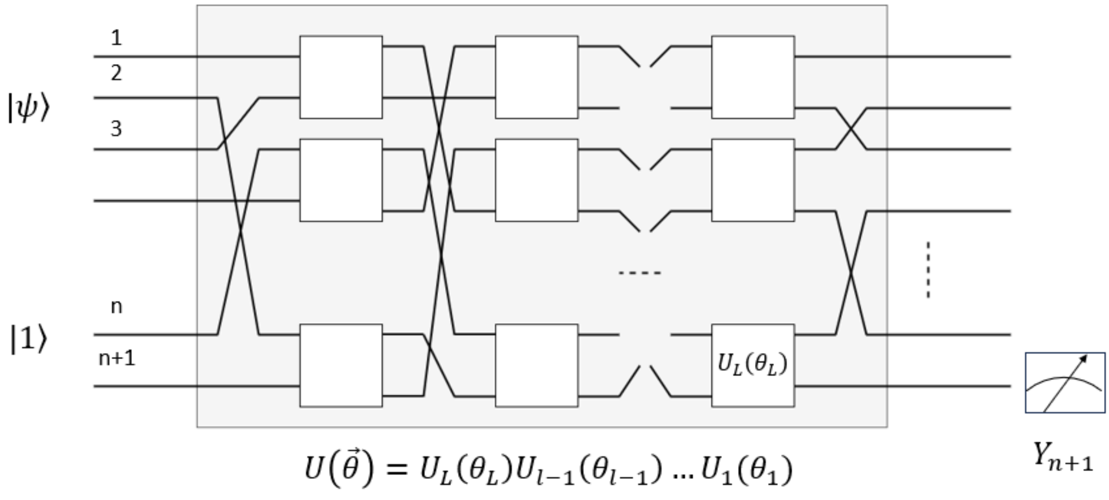

Figure 4. workflow of QNNs.

4.4. Quantum Support Vector Machines (QSVMs)

The conventional support vector machine (SVM) technique has a quantum version known as the Quantum Support Vector Machine (QSVM). Identifying the ideal hyperplane that divides several classes of data points is intended to categorize data. Calculating the kernel function in conventional support vector machines (SVM) increases computational cost with increasing dataset size or complexity. The kernel function calculates the similarity between data points. Using quantum concepts like superposition and entanglement, QSVM may compute the kernel function more quickly than classical SVMs, potentially providing exponential speedups for specific datasets. To improve the border between classes, QSVM performs quantum operations and encrypts data into quantum states. Its benefit comes from the quantum feature mapping method, which uses quantum circuits to convert the data into a higher-dimensional space. Because of this, QSVM can handle complicated datasets more effectively than traditional SVM, especially when the data cannot be separated linearly. QSVM has applications in several domains, including sentiment analysis, natural language processing (NLP), fraud detection, and picture classification. In these domains, it outperforms traditional SVMs regarding processing speed and accuracy. It can potentially resolve high-dimensional classification issues that standard algorithms cannot handle computationally.

The fundamental principle of quantum support vector machines (QSVM) is finding the best hyperplane to optimize the margin between classes. The traditional SVM technique resolves this by mapping the input data into a higher-dimensional space via the kernel function $K(x_i, x_j)$. In QSVM, using quantum operations in the kernel function computation provides potential speedups.

In QSVMs, the quantum kernel function $K_Q(x_i, x_j) = |\langle\psi(x_i)|\psi(x_j)\rangle|^2$ where $\psi(x_i)$ represents the quantum state corresponding to the classical input x_i, and $\langle\psi(x_i)|\psi(x_j)\rangle$ is the inner product of the quantum states. The goal of QSVM is to maximize the following objective function, similar to the classical SVM, as shown in Equation 9.

$$\min_{w} \frac{1}{2}||\omega||^2 + C\sum_{i=1}^{n} \max(0, 1 - y_i(\omega.\phi(x_i))) \quad (9)$$

where ω represents the weight vector, C is a regularization parameter, and $\phi(x_i)$ is the quantum feature mapping of the input data x_i.

The effective use of quantum circuits in the kernel function calculation provides the quantum advantage in QSVM. Compared to conventional approaches, these circuits enable kernel computation in fewer steps by encoding the data points as quantum states and carrying out operations, particularly for big and complicated datasets. An anti-noise quantum support vector machine technique was presented by Li et al. [76] to solve the problem of noisy data, which significantly impacts QSVM models' performance. In order to produce an anti-noise objective function, they included a weight component in the hinge loss function and created a quantum circuit to solve it. Their test findings showed consistent accuracy even under noisy data settings. Gentinetta et al. [77] presented a kernel-based quantum support vector machine that provides exponential speedups over classical techniques for specific datasets. The author observed how the probabilistic aspect of quantum mechanics affects the difficulty of QSVM training and, in some cases, offered a method that lessens the number of quantum circuit evaluations needed.

The Quantum Variational Kernel Support Vector Machine (QVK-SVM), which combines the advantages of quantum variational and quantum kernel algorithms, was first presented by Innan et al. [78]. Their research revealed that QVK-SVM performed more accurately than

Quantum Kernel SVM (QK-SVM) and Quantum Variational SVM (QV-SVM), offering a more reliable model for classification applications. Kavitha et al. [79] focused on choosing the best quantum feature map for certain benchmark datasets to increase the quantum support vector machines' execution speed and accuracy. Their findings showed that QSVM works faster and more accurately when dataset complexity rises than traditional SVM. Ruskanda et al. [80] used Quantum-Enhanced Support Vector Machines (QE-SVM) for natural language processing applications, namely sentiment analysis. They achieved excellent classification accuracy by optimizing circuit settings and data transformations, exceeding traditional SVM algorithms in their studies. Suzuki et al. [81] investigated the effectiveness of quantum support vector classification (QSVC) and quantum support vector regression (QSVR) models using a range of datasets, including tasks involving image classification and fraud detection. Their findings demonstrated that noiseless quantum circuit simulations can perform just as well as QSVC models on near-term quantum devices.

4.5. Grovers search algorithm

Grover's Search method is a renowned quantum method recognized for delivering a quadratic acceleration when searching an unsorted database. Classical search techniques often necessitate O(N) operations to locate an element inside a dataset of size N. However, Grover's algorithm lowers this cost to O(√N). The method functions by repetitively executing two primary operations: the oracle and the amplitude amplification, commonly called the Grover diffusion operator. The oracle identifies the accurate solution within the quantum superposition, and amplitude amplification enhances the likelihood of measuring the identified solution.

Grover's approach has many applications in domains involving search-related challenges, including cryptography, database retrieval, combinatorial optimization, and resolving NP-hard problems when integrated with other quantum methodologies, such as the Quantum Approximate Optimization approach (QAOA). Moreover, Grover's search technique is advantageous for identifying a single answer among several options, rendering it a vital tool for extensive unsorted data searches. Mathematically, Grover's algorithm is based on the quantum amplification process. Suppose the is to search for a solution x such that f(x)=1, where f(x) is an oracle function that identifies the solution. Grover's algorithm starts with a superposition over all possible states in the system, as shown in Equation 10.

$$|\boldsymbol{\psi_0}\rangle = \frac{\mathbf{1}}{\sqrt{\boldsymbol{N}}}\sum_{\boldsymbol{z}=\mathbf{0}}^{\boldsymbol{N}-\mathbf{1}} |\boldsymbol{x}\rangle \quad (10)$$

Oracle application: The Oracle marks the correct solution by flipping its phase, as shown in Equation 11.

$$\boldsymbol{O}|\boldsymbol{x}\rangle = (-\mathbf{1})^{f(x)}|\boldsymbol{x}\rangle \quad (11)$$

The phase is flipped for the correct solution, and it remains unchanged for all other states. After applying the oracle, Grover's algorithm performs an inversion about the mean operation, also known as the Grover diffusion operator, to amplify the probability amplitude of the marked solution, as shown in Equation 12.

$$\boldsymbol{D} = \mathbf{2}|\boldsymbol{\psi_0}\rangle\langle\boldsymbol{\psi_0}| - \boldsymbol{I} \quad (12)$$

where I is the identity operator. The overall process is repeated. $O(\sqrt{N})$ times to ensure that the amplitude of the correct solution approaches. After the appropriate number of iterations, the quantum state is measured, and the result will, with high probability, be the correct solution.

Jhanwar et al. [82] highlighted the promise of Grover's approach in machine learning, specifically for improving the efficiency of traditional machine learning tasks. They emphasized that Grover's technique can function as a subroutine to enhance search-related tasks in machine learning. Khanal et al. [83] investigated Grover's algorithm as a quantum classifier and showed its quadratic enhancement resembling conventional logic gates such as AND, XOR, and OR inside quantum circuits. Their tests confirmed the relevance of Grover's search for fundamental quantum machine learning problems. Ohno et al. [84] presented the Grover Learning Oracle (GLO) method, an adapted variant of Grover's search tailored for binary optimization challenges. GLO employed a learning-based oracle to iteratively identify solutions with fewer cost function evaluations than conventional Grover Adaptive Search (GAS). Tonchev et al. [85] investigated a quantum random walk variation of Grover's search employing modified Householder reflection and phase multipliers. Their Monte Carlo simulations and machine learning methodologies demonstrated that this adjustment conferred improved stability and resilience to variations in algorithm parameters.

4.6. Quantum Principal Component Analysis

Quantum Principal Component Analysis (QPCA) is a quantum algorithm that efficiently determines the principal components of a density matrix. It extends classical Principal Component Analysis (PCA) to quantum computing, utilizing the power of quantum superposition and entanglement to perform high-dimensional data analysis efficiently. Given a density matrix ρ of an n −qubit quantum system, the goal of QPCA is to extract the dominant eigenvectors corresponding to the largest eigenvalues. The eigenvalue decomposition ρ of is given in Equation 13.

$$\boldsymbol{\rho} = \sum_{i} \boldsymbol{\lambda}_i |\, \boldsymbol{v}_i\rangle\langle \boldsymbol{v}_i| \tag{13}$$

where λ_i are the eigenvalues and $|v_i\rangle$ are the corresponding eigenvectors. QPCA uses phase estimation to extract these eigenvalues efficiently. The QPCA algorithm effectively extracts eigenvalues by phase estimation. The steps include quantum phase estimation to estimate eigenvalues, eigenvalue thresholding to filter primary components, and state preparation to collapse the system to the dominant eigenvectors, as shown in Equation 14.

$$\boldsymbol{U_{PE}}|\boldsymbol{\psi}\rangle|\boldsymbol{0}\rangle = \sum_{i} \sqrt{\boldsymbol{\lambda}_i}|\boldsymbol{v}_i\rangle|\overline{\boldsymbol{\lambda}_i}\rangle \tag{14}$$

where U_{PE} is the quantum phase estimation unitary, $|\psi\rangle$ is the input quantum state, and $|\overline{\lambda_i}\rangle$ is the estimated eigenvalue state. He et al. [86] provided a low-complexity quantum principal component analysis (qPCA) technique. Similar to state-of-the-art qPCA, it accomplishes dimension reduction by extracting the main components of the data matrix, rather than all components of the data matrix, to quantum registers, resulting in significantly fewer measurement samples. The qPCA and Lin's qPCA used quantum singular-value thresholding (QSVT). Lin's qPCA combines QSVT and modified QSVT to achieve principal component superposition. Lin et al. [87] presented an improved quantum principal component analysis (Improved qPCA) approach that uses a fixed threshold. The author may reduce the singular value less than the threshold to zero to produce a target quantum state, which can then be utilized to provide an output similar to qPCA following phase estimation. Compared to qPCA, the proposed method only considered the target eigenvalues and the probability of obtaining each eigenvalue is greater. Despite its potential, QPCA has various drawbacks, the most significant of which are the need for fault-tolerant quantum computers and controlled-unitary operations, both of which are difficult to execute. However, recent advances in hybrid quantum-classical approaches have yielded approximation PCA solutions, and quantum embedding techniques in machine learning are improving dimensionality reduction methods.

4.7. Quantum K-means clustering

Quantum K-Means is a quantum-enhanced variant of the traditional K-Means clustering technique that divides a dataset into clusters more efficiently. It uses quantum computing for distance computations and cluster assignment, resulting in greatly improved computational efficiency. In traditional K-Means, the cost function is reduced to maximize clustering, which involves iterating over data points to determine the best centroids. Equation 15 presents the mathematical definition of the cost function.

$$\boldsymbol{J} = \sum_{\boldsymbol{i=1}}^{\boldsymbol{k}} \sum_{\boldsymbol{x \in C_i}} ||\boldsymbol{x} - \boldsymbol{\mu}_i||^2 \tag{15}$$

where J represents the total cost, k is the number of clusters, x are data points, and μ_i are the cluster centroids. The quantum method speeds up this process by utilizing quantum distance estimation techniques to effectively compute Euclidean distance. The system encodes distances to numerous centroids at the same time, which simplifies cluster assignment. Furthermore, Grover's search strategy helps to optimize centroid selection, as demonstrated in Equation 16.

$$||\boldsymbol{x} - \boldsymbol{y}||^2 = \langle \boldsymbol{x}|\boldsymbol{x}\rangle + \langle \boldsymbol{y}|\boldsymbol{y}\rangle - 2\langle \boldsymbol{x}|\boldsymbol{y}\rangle \tag{16}$$

where x and y are the data points and $\langle x|y\rangle$ represents the quantum inner product operation. However, the actual implementation of Quantum K-Means is limited by existing quantum hardware constraints, such as qubit count and noise interference. Despite these challenges, hybrid quantum-classical clustering algorithms are becoming beneficial, especially for high-dimensional clustering workloads. Researchers aim to make quantum clustering methods more feasible by combining quantum embeddings with classical machine learning approaches.
Poggiali et al. [88] developed, built, and analyzed three hybrid quantum k-Means algorithms with different degrees of parallelism. Indeed, each method gradually employed quantum parallelism to decrease the complexity of the cluster assignment phase to a fixed cost. The author uses quantum phenomena to accelerate distance computations. The essential notion is that the distances between records and centroids may be calculated concurrently, reducing time, particularly for large datasets. Meshram et al. [89] described the latest advancements in the QPSO-k-means clustering method, emphasizing swarm initialization and algorithm parameter optimization. The author verified the approach on the UCI healthcare dataset and showed that it can solve inefficient clustering by adjusting parameters like the number of iterations, error rate, and optimal solution for cluster centers.

4.8. Comparative Analysis of QML Algorithms

This section presents the results of the comparative analysis conducted to evaluate different Quantum Machine Learning optimization algorithms. The analysis is based on the systematic review of selected studies as described in the research methodology. The purpose of this analysis is to answer the research objective by identifying the performance, strengths and limitations of various QML algorithms. The results show that QAOA and VQE

Table 2. SWOT analysis of existing models.

Model	Strengths	Weaknesses	Opportunities	Threats
Quantum Approximate Optimization Algorithm (QAOA)	Suitable for combination optimization, hybrid quantum-classical approach, near-term implement ability	Performance highly, dependent on parameter tuning, limited by hardware noise	Applications in logistics, finance, and AI	Scalability issues, requires high-quality quantum hardware
Variational Quantum Eigensolver (VQE)	Effective for quantum chemistry and material simulations, adaptable to near-term quantum devices	Sensitive to noise, requires a classical optimizer for parameter updates	Drug discovery, energy optimization, material science applications	Hardware constraints, classical simulation sometimes competitive
Quantum Neural Networks (QNNs)	Potential for speedup in AI/ML tasks, improved pattern recognition capabilities	Training quantum network is complex, lack of standardized architectures	Future AI advancements, quantum-enhanced deep learning models	Lack of hardware stability, classical AI is still evolving rapidly
Quantum Support Vector Machines (QSVMs)	Can leverage quantum speedup for classification tasks, beneficial for large datasets	Requires large, high-quality qubits, not fully practical yet	Revolutionizing machine learning and data analysis	Competing classical ML methods, slow adoption in the industry
Quantum Principal Component Analysis (QPCA)	Exponential speedup in high-dimensional data analysis, potential improvements in feature extraction	Difficult to implement on current hardware, requires quantum memory	Transforming big data analysis and AI applications	Competing classical PCA methods, quantum error correction challenges
Grover's search algorithm	Quadratic speedup over classical search algorithms, fundamental for unstructured data searches	Requires fault-tolerant quantum computers, limited to specific tasks	Speeding up database searches, cryptographic applications	Development of better classical heuristics, large qubit requirements
Quantum k-means	Faster clustering of large datasets, better pattern recognition	High error rates in practical quantum computations, limited qubit connectivity	Advancing data science, quantum-enhanced clustering AI	Limited scalability, classical clustering algorithms improving rapidly

are highly effective for optimization problems, particularly in combinatorial and energy optimization tasks. Similarly, Quantum Neural Networks and Quantum Support Vector Machines demonstrate strong performance in learning-based optimization. However, the analysis also reveals limitations such as scalability constraints, hardware noise, and implementation complexity. these findings represent the net results of the data analysis and provide insight into the practical applicability of Quantum Machine Learning for optimization.

Quantum Machine Learning (QML) algorithms enable significant speedups over traditional approaches, but their strengths and shortcomings vary depending on the problem domain. The Quantum Approximate Optimization Algorithm (QAOA) is especially useful for combinatorial optimization problems, providing polynomial speedup while being compatible with noisy quantum devices. The Variational Quantum Eigen solver (VQE) accelerates quantum chemistry and material science problems exponentially, but it needs near-term quantum devices that are generally noiseless. Quantum Principal Component Analysis (QPCA) provides an exponential advantage in dimensionality reduction and quantum data processing but requires fault-tolerant quantum devices. Meanwhile, Quantum K-Means achieves polynomial speedups in clustering tasks, making it especially valuable for machine learning and image segmentation.

Despite their advantages, QML algorithms confront noise sensitivity and scalability challenges. Algorithms like QPCA and VQE need high-fidelity qubits, but QAOA and Quantum K-Means are more suited to near-term quantum devices. To overcome the theoretical and practical gap, researchers are working on hybrid quantum-classical models that combine conventional optimization approaches with quantum processing speed up. Equation 17 summarizes the mathematical comparison between these methods.

Table 3. Performance metrics of classical vs quantum methods.

Metric	Classical method	Quantum method
Computational speed	Polynomial or exponential time complexity (depending on the problem)	Potential for exponential speedup (e.g., Grover's algorithm)
Scalability	Limited by Moore's law and classical hardware improvements	Highly scalable with qubit improvements but limited by decoherence
Accuracy	High accuracy for well-established classical algorithms	Can outperform classical methods in some tasks but sensitive to noise
Hardware efficiency	Relies on classical processors (CPU, GPU, TPU)	Requires specialized quantum hardware (superconducting, trapped ions)
Memory requirements	High memory usage for large-scale problems	Requires quantum memory, which is still under development
Optimization problems	Classical solvers (e.g., simulated annealing, gradient descent)	Quantum algorithms like QAOA and VQE show potential improvements
Data processing	Handles structured and unstructured data efficiently	Best suited for complex, high dimensional data processing
Energy consumption	High for classical supercomputers	Lower power consumption but requires cryogenic cooling
Error sensitivity	Robust error correction in classical computing	High error rates, needs fault-tolerant quantum computing
Practical implementation	Widely used across industries	Still in experimental and early adoption stages

$$\boldsymbol{S_{QML}} = \sum_{i} \boldsymbol{\alpha_i A_i} + \boldsymbol{\beta_i B_i} \tag{17}$$

where S_{QML} represents the performance matrix, α_i and β_i are weighting factors and A_i and B_i are algorithm-specific performance indicators. Such integrations enhance the feasibility of QML in real-world applications, ensuring that quantum computing advances in optimization, clustering, and data analysis domains. Unlike classical optimization methods, QML algorithms provide enhanced capability to handle complex and high-dimensional optimization problems. However, this review critically identifies that current QML algorithms still face practical limitations, including hardware constraints, noise sensitivity and scalable challenges. The novelty of this review lies in presenting a critical comparison between different QML optimization approaches of this review lies in presenting a critical comparison between different QML optimization approaches while clearly highlighting their advantages, limitations, and practical applicability. This analysis provides important insights for researchers to develop more efficient and scalable quantum optimization solutions. Table 2 shows a SWOT analysis. Performance metrics of classical vs quantum methods are shown in Table 3.

5. Challenges in quantum machine learning for optimization

Quantum Machine Learning (QML) offers significant potential for addressing complex optimization challenges using quantum computing capabilities. Nonetheless, specific challenges hinder the scalability, precision, and feasibility of Quantum Machine Learning (QML) algorithms, particularly on existing Noisy Intermediate-Scale Quantum (NISQ) devices. Challenges and future directions are shown in Table 4. The following are significant issues faced with QML for optimization:

5.1. Scalability and Qubit Limitations

A primary problem in QML is scalability, mainly due to current quantum technology constraints. Quantum algorithms sometimes need substantial quantities of qubits and complex quantum circuits to address complicated optimization challenges effectively (Abbas et al. [90]). However, contemporary quantum computers are constrained by the number of qubits and their coherence durations. Currently, most NISQ devices accommodate just a few hundred qubits, with qubits extremely vulnerable to noise and decoherence. Consequently, scaling QML algorithms to address substantial, real-world optimization challenges is challenging, as both the qubit count and circuit depth directly influence the algorithm's efficacy and precision (Ajagekar et al. [91]).

Moreover, the physical implementation of qubits in quantum devices necessitates meticulous management and error correction, constraining the number of qubits that may be effectively utilized in quantum optimization tasks (Bharti et al. [92]). In the absence of substantial advancements in quantum hardware, such as the creation of error-corrected qubits and quantum processors with enhanced qubit coherence times, the scalability of Quantum Machine Learning will remain limited.

5.2. Noise and error rates

A significant challenge in QML is the presence of noise and gate errors in existing quantum hardware. Qubits exhibit high sensitivity to their surroundings, resulting in decoherence, information loss, and mistakes in gate operations throughout quantum computations. These errors notably impact the performance of QML algorithms (Bharti et al. [92]), particularly in iterative processes like the Quantum Approximate Optimization Algorithm (QAOA) and the Variational Quantum Eigensolver (VQE).

The interference caused by noise in quantum gates leads to incorrect transformations of quantum states, resulting in suboptimal optimization outcomes. Although traditional error correction methods are effective and commonly implemented, quantum error correction necessitates significant overhead regarding extra qubits and computational resources. The buildup of errors in quantum gates within QML diminishes the likelihood of achieving precise outcomes, thereby constraining the real-world use of quantum optimization techniques on noise-affected hardware. Furthermore, quantum error correction codes, including surface codes, necessitate using hundreds of physical qubits to represent a single logical qubit, thereby contributing to the hardware's constraints. Consequently, identifying efficient methods to minimize noise in quantum systems and developing noise-resistant QML algorithms are essential for achieving dependable optimization outcomes.

5.3. Algorithm design and hybrid models

Hybrid quantum-classical systems represent a promising avenue in quantum machine learning today, merging the capabilities of quantum computations with the reliability and efficiency of classical optimization techniques (McClean et al. [93]). Algorithms like QAOA and VQE function by alternating between quantum and classical processes, with quantum circuits producing solutions and classical optimizers fine-tuning the parameters. Nonetheless, the architecture of hybrid quantum-classical algorithms brings forth further intricacies.

One significant challenge is enhancing the interaction between the quantum and classical components. The quantum circuit assesses a cost function forwarded to the classical optimizer (Endo et al. [94]). This procedure needs to be executed several times to reach an optimal solution, which increases the number of iterations and computational demands. The delay and communication burden between quantum and classical systems can hinder the overall optimization process, resulting in a level of efficiency that falls short of expectations.

Moreover, choosing suitable quantum ansatzes (parameterized quantum circuits) in hybrid models significantly impacts the algorithm's performance. An inadequately constructed ansatz may lead to sluggish convergence or an inability to identify optimal solutions (Barchielli et al. [95]). With advancements in quantum hardware, it will be crucial to enhance the algorithmic interface and design of ansatz to maximize the effectiveness of hybrid models in addressing optimization challenges.

5.4. Overfitting and Model Generalization

In machine learning, overfitting happens when a model excels at the training data but struggles to apply its knowledge to new, unseen data. This issue holds significance in quantum machine learning, particularly concerning quantum neural networks and various quantum models trained using noisy or restricted datasets. Quantum models often exhibit overfitting in scenarios where data is limited or when the complexity of the model (Peters et al. [96]), such as the number of qubits or quantum gates, is excessively high. The limited size of datasets commonly employed in quantum experiments, coupled with the intrinsic noise present in quantum systems, makes the challenge of model generalization particularly significant. Quantum models might excel on the training dataset yet face challenges with new instances because they fail to grasp the fundamental patterns present in the data. Moreover, the vulnerability of quantum systems to noise intensifies the overfitting issue, as noisy quantum circuits might unintentionally "learn" patterns from random fluctuations instead of significant data characteristics (Banchi et al. [97]).

To address these challenges, regularization techniques from classical machine learning, like dropout or weight decay, must be adapted for application in quantum systems. Furthermore, employing techniques such as cross-validation and meticulously adjusting hyperparameters is essential for guaranteeing that QML models perform effectively on new, unseen data.

6. Use Cases and Real-World Applications of Quantum Machine Learning (QML)

Quantum Machine Learning (QML) presents significant opportunities in numerous practical applications where traditional methods face computational complexity, data volume, and scalability challenges. Utilizing the distinctive characteristics of quantum computing, QML can greatly improve optimization strategies in various fields. Here are three significant use cases that illustrate the practical impact of QML:

6.1. Portfolio optimization in finance

Portfolio optimization represents a core challenge in finance, as investors aim to distribute assets to achieve the highest returns with the least risk. Traditional optimization techniques, like Markowitz's mean-variance optimization, face challenges with extensive, intricate portfolios, particularly when navigating the uncertainties inherent in financial markets (Kanaparthi et al. [98]). Quantum-enhanced portfolio optimization presents innovative solutions by effectively navigating the extensive landscape of potential asset allocations and pinpointing optimal portfolios more rapidly than traditional algorithms (Jha et al. [99]).

QML, especially algorithms such as the Quantum Approximate Optimization Algorithm (QAOA) and Variational Quantum Eigensolver (VQE), can be utilized to address the portfolio optimization problem by framing the asset allocation issue as a combinatorial optimization challenge. These algorithms facilitate the parallel exploration of extensive solution spaces, leading to a considerable decrease in the time needed to identify the optimal allocation. For instance, Grover's algorithm can be utilized to identify the optimal portfolio by exploring unsorted datasets more efficiently than traditional search techniques. In finance, QML helps in risk management, asset allocation, and option pricing, presenting the opportunity to surpass traditional models in speed and accuracy (Mironowicz et al. [100]), especially when addressing complex portfolios that necessitate the evaluation of various factors such as risk, return, and liquidity.

6.2. Supply chain and route optimization in logistics

Optimizing supply chains and efficiently routing goods present significant challenges for businesses globally. These challenges entail addressing intricate combinatorial optimization tasks, including identifying the most efficient delivery routes for goods (routing problems) or reducing costs and delays within supply chains (inventory optimization). Conventional optimization techniques, such as linear programming or genetic algorithms, frequently encounter challenges when addressing the scale and complexity of practical logistics issues (Chen et al. [101]).

Quantum Machine Learning (QML) provides practical solutions for optimizing supply chains and routes, significantly improving the efficiency of these operations. Algorithms such as QAOA can address routing challenges, including the Traveling Salesman Problem (TSP) (Qian et al. [102]), by encoding the optimization task within a quantum circuit and harnessing quantum parallelism to investigate various routes simultaneously. This facilitates quicker and more effective optimization of smart cities' delivery schedules, inventory management, and vehicle routing.

QML can be utilized in inventory management, enhancing stock levels to minimize expenses while maintaining product availability. By examining extensive datasets related to customer demand, warehouse placements, and delivery timelines, QML can offer optimal computationally unfeasible solutions for traditional approaches, allowing businesses to enhance their supply chains and lower operational expenses.

6.3. Molecular optimization in drug discovery

In the pharmaceutical sector, a highly resource-intensive endeavor involves simulating molecular interactions to identify novel drug candidates. Traditional computing systems encounter considerable obstacles in simulating the quantum mechanical behavior of molecules, as the computational demands increase exponentially with the number of particles involved. This limits the capacity to investigate possible pharmaceuticals and refine their molecular configurations effectively. Quantum Machine Learning (QML), mainly via algorithms such as VQE, can enhance molecular optimization by simulating the quantum states of molecules with greater efficiency compared to classical approaches. VQE estimates molecules' ground state energies, which are essential for comprehending molecular stability and reactivity. Utilizing quantum computers to simulate molecular interactions enables the exploration of larger and more complex molecules, which could result in the discovery of new drugs. QML can help in protein folding and ligand binding simulations, which are essential drug development components involving intricate molecular optimizations (Li et al. [76]). In these scenarios, QML allows for simultaneously simulating thousands of molecular conformations, pinpointing the optimal configuration for drug candidates. This process effectively shortens the time needed for drug discovery and lowers the costs tied to experimental trials.

7. Conclusion

Quantum Machine Learning (QML) has emerged as a promising frontier, combining the power of quantum computing with the flexibility of machine learning. This research survey analyzed the role of QML in solving complex optimization problems through a comprehensive review of existing algorithms and applications. The findings confirm that QML algorithms such as QAOA, VQE, QSVMs, QNNs, and quantum clustering methods demonstrate significant potential in improving optimization efficiency compared to classical methods, particularly for complex and high-dimensional problems. Based on the analysis, this study highlights that despite promising performance, practical implementation of QML is still limited by hardware constraints, scalability issues and quantum noise. These findings clearly answer the research objective by confirming the effectiveness as well as limitations of QML in optimization. The main implication of this research is that hybrid quantum-classical optimization frameworks can provide a practical for real-world optimization problems until fully scalable quantum hardware becomes available. Furthermore, this study contributes a conceptual insight that future optimization systems will likely integrate quantum learning models with classical computing to achieve optimal performance.

Table 4. Challenges and future direction of QML.

Challenge	Description	Future direction
Scalability	Quantum hardware is still limited	Improve qubit coherence
Noise sensitivity	QML algorithms suffer from noise interference	Advanced error correction
Hybrid integration	Classical-quantum synergy is needed	Efficient hybrid models
Algorithm optimization	Better quantum algorithms are required	Research on new quantum heuristics

This research provides important guidance for future researchers to focus on developing scalable, noise-resilient, and practically implementable QML optimization models for real-world applications.

8. Future work

The future of Quantum Machine Learning (QML) is strongly dependent on advances in quantum hardware, error correction, and algorithmic development. Emerging qubit technologies, such as superconducting qubits and topological qubits, seek to address present scaling concerns, allowing for more powerful quantum processors. Simultaneously, research on fault tolerance and error mitigation is critical to enhancing the dependability of quantum computers. Creating unique quantum algorithms specialized for certain industries, such as banking, healthcare, and logistics, will help to improve optimization capabilities. Furthermore, hybrid quantum-classical integration remains a top priority, since improved frameworks for mixing quantum acceleration with conventional processing will drive real-world applications. Continued work in these areas will accelerate the shift from theoretical promise to practical effect, influencing the next phase of quantum computing.

9. List of Abbreviations

Abbreviation	Full Form
QML	Quantum Machine Learning
QAOA	Quantum Approximate Optimization Algorithm
VQE	Variational Quantum Eigensolver
QNN	Quantum Neural Network
QSVM	Quantum Support Vector Machine
ML	Machine Learning
AI	Artificial Intelligence
NISQ	Noisy Intermediate-Scale Quantum
NLP	Natural Language Processing
NP	Non-deterministic Polynomial
SVM	Support Vector Machine

10. Declarations

10.1. Author Contributions

Uzma Nawaz: Conceptualization, Methodology, Formal analysis, Investigation, Data Curation, Writing – Original Draft, Visualization. **Zubair Saeed:** Supervision, Validation, Writing – Review & Editing, Conceptual guidance, Critical revision of the manuscript. **Kamran Atif:** Supervision, Project administration, technical guidance, Writing – Review & Editing, Final approval of the manuscript.

10.2. Institutional Review Board Statement

Not applicable.

10.3. Informed Consent Statement

Not applicable.

10.4. Data Availability Statement

This study is based on a systematic review of previously published research articles. All data used in this study are available in the public domain through the cited references.

10.5. Acknowledgment
Not applicable.

10.6. Conflicts of Interest
The authors declare no conflicts of interest.

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