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Article

# Brine Treatment Plant using Hybrid Forward Osmosis–Membrane Distillation (FO–MD) System

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**Abstract:** Brine discharge from seawater reverse osmosis (SWRO) plants poses critical environ-mental and operational challenges, particularly in regions reliant on large-scale desalination. This study proposes a hybrid brine treatment system integrating Forward Osmosis (FO) and Membrane Distillation (MD) to enhance water recovery and minimize ecological impact. The FO stage utilizes a concentrated magnesium chloride ($MgCl_2$) draw solution to extract water from high-salinity brine without the need for hydraulic pressure, while the MD stage regenerates the draw solution using low-grade solar thermal energy, simultaneously producing high-purity distillate. Mass and energy balance calculations were per-formed to evaluate recovery rates, specific energy consumption, and thermal input requirements. The results indicate that the FO–MD configuration can achieve recovery rates exceeding 80% with significantly reduced brine discharge, while maintaining low energy demand compared to conventional methods. The integration of solar energy further enhances system sustainability, making it suitable for deployment in off-grid or arid regions. This hybrid approach demonstrates strong potential for advancing sustainable desalination practices, aligning with circular water strategies and zero liquid discharge (ZLD) objectives.

**Keywords:** Forward Osmosis; Membrane Distillation; Brine Treatment; Hybrid Desalination; Renewable Energy; Zero Liquid Discharge.



## 1. Introduction

As global reliance on seawater desalination grows, particularly through reverse osmosis (RO), the management of the resulting high-salinity brine has become an increasingly critical challenge. Brine dis-charge from RO systems poses serious environmental threats to marine ecosystems due to its high salt con-centration and chemical additives, while conventional disposal methods such as deep-well injection or direct discharge into the sea are both energy-intensive and unsustainable. Existing treatment technologies are of-ten costly, complex, and reliant on high-grade energy sources, making them unsuitable for decentralized or resource-limited applications. To address these challenges, this project proposes a standalone hybrid brine treatment system that integrates Forward Osmo-sis (FO) and Membrane Distillation (MD), operating primarily on low-grade solar thermal energy. The FO–MD configuration provides an energy-efficient, modular, and compact solution that enhances freshwater recovery while reducing the environmental foot-print of brine discharge. Designed for integration with existing desalination facilities or deployment in remote and off-grid locations, the system offers a scalable and sustainable pathway for modern brine management in line with global and regional sustainability goals [1].

Desalination facilities remove dissolved salts and minerals from seawater to produce freshwater suitable for drinking. This technology plays a crucial role in supplying water to arid regions and island countries that face scarcity of natural freshwater resources.

The general stages of a seawater desalination process, including intake, pretreatment, reverse osmosis, and brine disposal, are shown in Figure 1. Continuous maintenance and efficient operation of seawater desalination plants are essential to ensure high-quality drinking water production and maintain system efficiency [2].

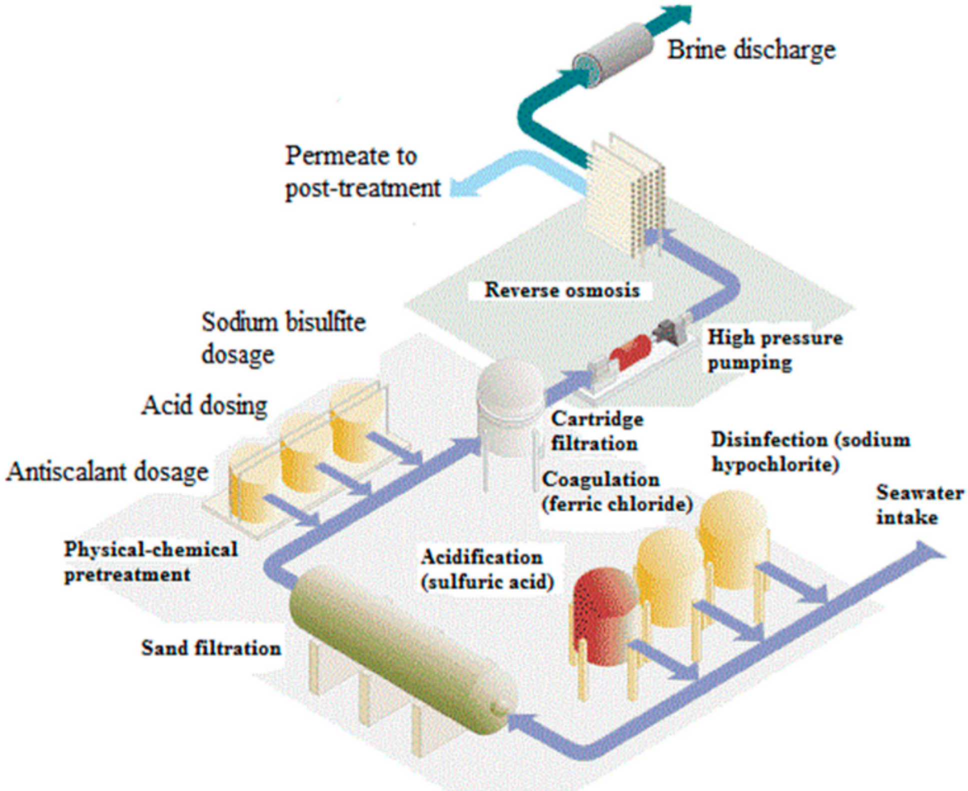


**Figure 1.** SWRO Process Overview [2].

The main novelty of the presented study is the development of a hybrid FO-MD system for the treatment of brine from desalination plants. The system aims to efficiently recover water from brine, reducing its environmental impact. The brine generated from seawater reverse osmosis (SWRO) desalination contains high concentrations of various salts and minerals, including sodium ($Na^+$), chloride ($Cl^-$), magnesium ($Mg^{2+}$), calcium ($Ca^{2+}$), sulfate ($SO_4^{2-}$), potassium ($K^+$), bromide ($Br^-$), boron (B), lithium (Li), rubidium (Rb), and strontium (Sr) [3]. While the current study focuses on water recovery using the hybrid FO–MD system, it should be noted that valuable minerals can potentially be extracted from the residual brine, offering an opportunity for future valorization and resource recovery.

This paper is organized as follows: Section 2 presents a detailed literature review on Forward Osmosis (FO) and Membrane Distillation (MD), and hybrid desalination systems, emphasizing their operational principles and environmental implications. Section 3 outlines the detailed methodology of the proposed hybrid FO–MD system, including process design, operational parameters, and the Mathematical calculations used for performance evaluation. Section 4 presents and discusses the results obtained from the system analysis, focusing on recovery rates, energy efficiency, and environmental benefits compared to conventional desalination methods. Section 5 summarizes the main conclusions and recommendations for future research and large-scale implementation.

## 2. Literature Review

Forward osmosis is a process in which a semi-permeable membrane divides two solutions of unequal solute concentrations, allowing solvent molecules to diffuse from the lower concentration solution to the higher one under the influence of osmotic pressur [4].

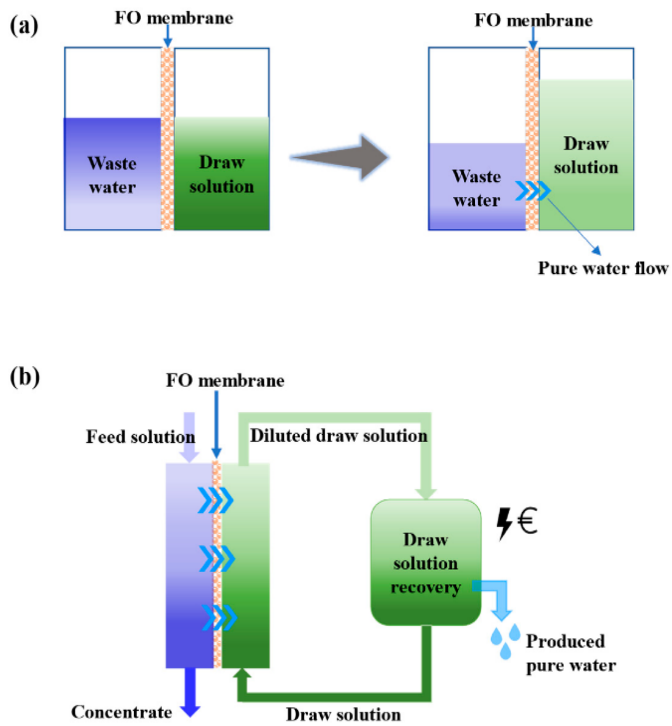


**Figure 2.** Forward osmosis [4].

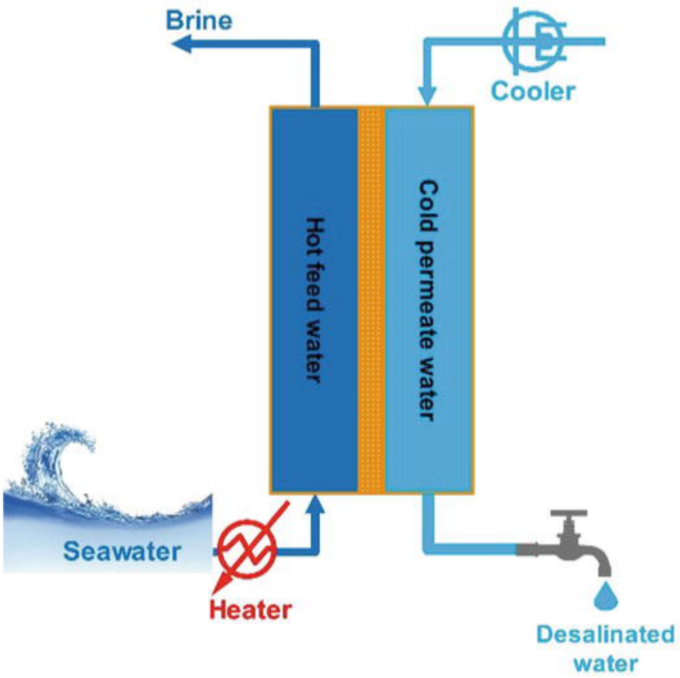


**Figure 3.** Operating Principle of MD [5].

2.1. Principles Forward osmosis (FO)

Forward osmosis (FO) is a membrane-based water separation technique that employs a semi-permeable membrane to extract water from a solution containing dissolved solutes. The process utilizes the natural osmotic pressure difference between two solutions to drive the movement of water molecules through the membrane, while preventing solute transfer.

In an FO system, the feed solution which has a lower solute concentration flows along one side of the membrane, whereas the draw solution, characterized by a higher solute concentration, passes along the opposite side. The osmotic gradient naturally drives water from the feed side toward the draw side through the membrane. As this occurs, the draw solution becomes more diluted, while the feed solution becomes increasingly concentrated, as depicted in Figure 2 [4].

2.2. Principles of MD

MD is a thermally-driven membrane separation process, only vapor molecules transfer across a microporous hydrophobic membrane [6]. The driving force during MD process is the difference between A temperature gradient between the two sides of the membrane creates a vapor pressure difference by circulating the hot feed solution (FS) and the cold permeate stream. Water vapor produced from the heated feed passes through the membrane and condenses on the cooler side, forming the distillate. The vapor pressure difference becomes greater as the temperature gap between the feed and permeate increases.One of the main benefits of membrane distillation (MD) is its ability to operate effectively at relatively low temperatures, generally below 80°C [7]. Figure 3 illustrates the working principle of the MD process for seawater desalination to obtain fresh water.In MD, the hydrophobic membrane structure allows vapor molecules from the hot feed to move through its pores while preventing the liquid phase from entering them due to the membrane's low surface energy [8]. Consequently, liquid–vapor interfaces form at the openings of the hydrophobic pores. To ensure proper operation, the membrane pores must remain dry, preventing liquid water from penetrating through the hydrophobic layer [5].

2.3. Hybrid SWRO–MD–FO Systems

In recent studies on hybrid brine treatment systems, Son et al. (2024) conducted an experimental investigation of a SWRO–MD–FO system for seawater brine management. The study showed that water recovery from the MD process is limited to less than 10% within the practical operating range, with fouling control being essential due to calcium sulfate and calcium carbonate scaling. Scaling was managed either by operating below solubility limits or using antiscalants. The Volumetric Concentration Factor (VCF) was employed as a key variable to define operating conditions, and experiments were performed using actual seawater and SWRO brine from a full-scale plant at KAUST, Saudi Arabia [1].

2.4. Desalination brine and its environmental impacts

Brine also referred to as concentrate or reject is the concentrated saline effluent generated as a by-product of desalination, as shown in Figure 4. This stream typically contains the majority of the dissolved solids from the feed water in a concentrated form, along with traces of pretreatment chemicals such as antiscalants, coagulants, flocculants, and microbial contaminants.The term brine broadly describes the saline by-product of desalination processes, irrespective of its exact salinity level. However, it is most commonly used to describe saline streams with total dissolved solids (TDS) exceeding 55,000 mg/L, which are categorized as high-TDS brine. The characteristics and volume of desalination brine are

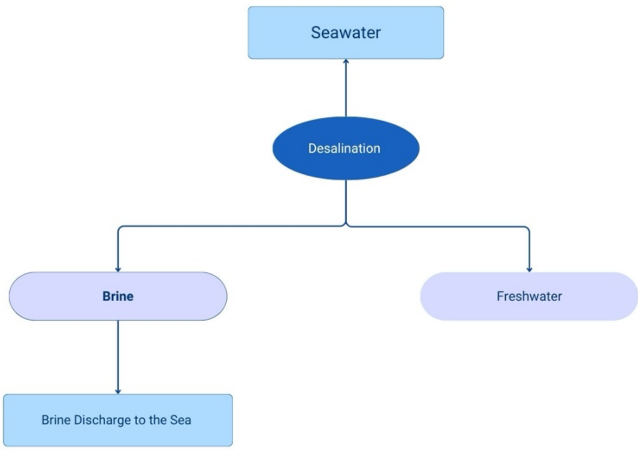


**Figure 4.** Brine Disposal.

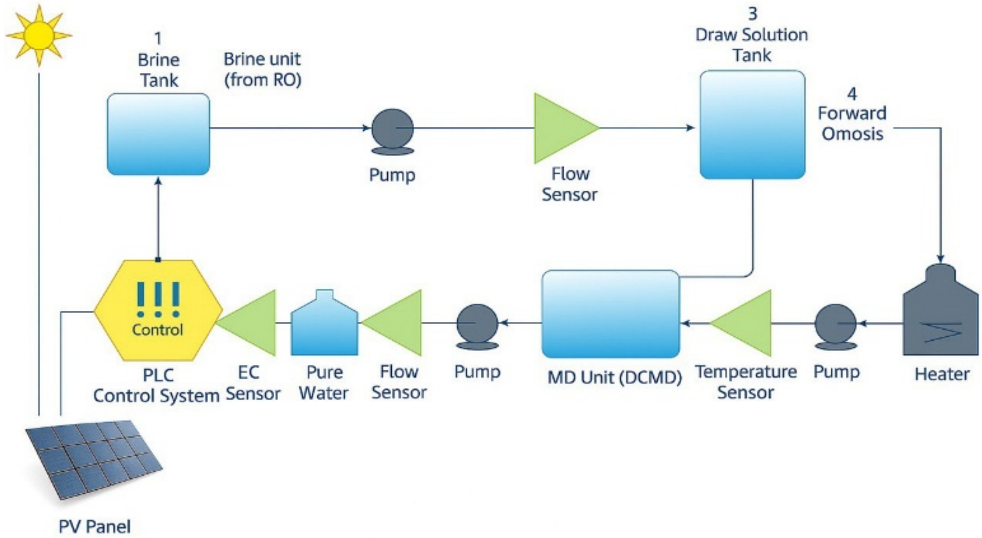


**Figure 5.** Brine treatment layout.

influenced by several factors, including feed water composition, pretreatment procedures, the type of desalination process, and the water recovery ratio [9].

2.5. PV–RO desalination

Janowitz et al. (2025) studied PV-powered seawater desalination using reverse osmosis under variable solar power. They showed that adjusting the operation to match solar availability can maintain stable performance and water quality, supporting decentralized or pilot-scale applications [10].

2.6. FO–MD Hybrid System

Kamel et al. (2023) indicated that the FO–MD hybrid system can efficiently treat seawater, brackish water, and wastewater while reducing energy consumption and brine disposal. The study addressed membrane performance, fouling, concentration polarization, and reverse solute diffusion, highlighting strategies to improve water recovery. The FO–MD system was emphasized as a sustainable solution compatible with renewable energy sources and applicable for water desalination, wastewater treatment, and resource recovery [11]. Additionally, Lee et al. (2020) demonstrated that in the integrated FO–MD hybrid system, the FO flux is influenced by the flow rates of the feed and draw solutions. An increase in the feed flow rate slightly improves FO flux by reducing external concentration polarization (ECP), whereas an increase in draw solution flow rate may slightly reduce FO flux due to internal concentration polarization (ICP) effects. Therefore, FO flux cannot be fully controlled by flow rates alone [12].

**3. Methodology**

The proposed system is based on a hybrid Forward Osmosis (FO) and Membrane Distillation (MD) configuration designed to treat high-salinity brine in an energy-efficient and sustainable manner. The system aims to recover water effectively while minimizing environmental impacts. Water from Reverse Osmosis (RO) brine is first transferred to a concentrated magnesium chloride ($MgCl_2$) draw solution through FO, after which the diluted draw solution is processed in MD to produce high-purity water. The residual brine remains concentrated but is not the focus of this study.

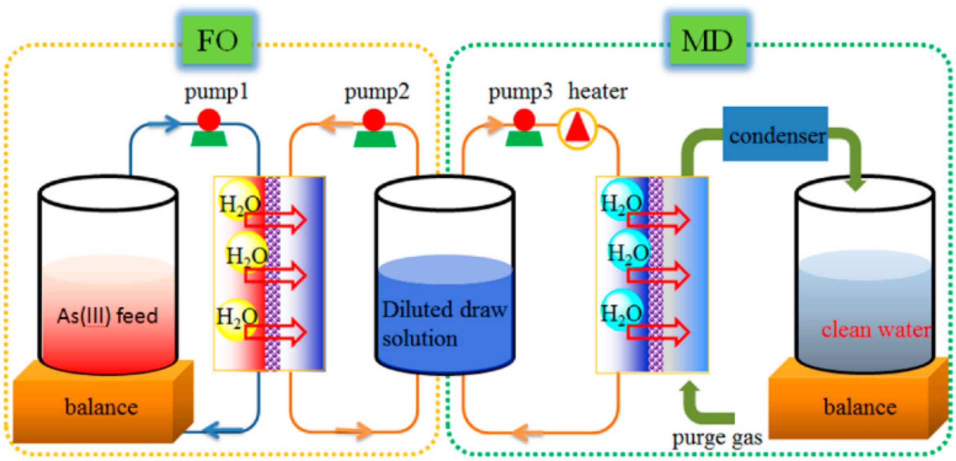


**Figure 6.** Schematic Diagram of the Hybrid FO–MD Process for Wastewater Treatment [13].

As shown in Figure 5, the control system monitors temperature, flow rate, and conductivity using sensors to maintain optimal FO–MD performance. The heater temperature setpoint is 60°C, with power adjusted based on real-time temperature and solar energy availability to optimize heating. Flow rates for feed and permeate streams are controlled at 1 $m^3$/h to sustain a stable temperature gradient and maximize water recovery Conductivity is kept below 20 μS/cm by adjusting flow rates or draw solution concentration as needed. Sensor data is continuously collected, enabling rapid controller response to maintain steady operation and water quality.

### 3.1. forward Osmosis (FO) Stage

In the first stage, Forward Osmosis (FO) is used to extract water from RO brine into a concentrated $MgCl_2$ draw solution. FO is a non-pressure-driven membrane separation process in which water passes through a semi-permeable membrane, water moves from the feed solution with lower osmotic pressure toward the draw solution with higher osmotic pressure. Since no hydraulic pressure is applied, FO has low energy consumption [14]. $MgCl_2$ is selected as the draw solute due to its high osmotic pressure, low reverse salt flux, chemical stability, and wide availability, which enhance water flux, reduce scaling, and support large-scale brine treatment. A Thin Film Composite (TFC) FO membrane is used for its high water permeability and chemical stability, ensuring efficient water transfer while rejecting most salts and contaminants.

### 3.2. Membrane Distillation (MD) Stage

In the second stage, Membrane Distillation (MD) is used to recover high-purity water from the diluted draw solution. MD is a thermally driven desalination process that employs a hydrophobic membrane acting as a physical barrier between the hot feed and the dis-tillate side, allowing only water vapor to pass while excluding the liquid feed. The vapor diffuses through the membrane due to a vapor pressure difference, condenses on the cold side, and produces high-quality distillate. The process involves heating the feed solu-tion, and the hydrophobic membrane prevents direct passage of salts and other non-volatile contaminants [15].

Based on the same principle, several MD configura-tions are available, including Direct Contact Mem-brane Distillation (DCMD), Air Gap MD (AGMD), Sweeping Gas MD (SGMD), and Vacuum MD (VMD). DCMD offers the simplest design, where both the feed and permeate are in direct contact with the membrane. VMD applies vacuum to reduce pres-sure on the permeate side, with vapors condensed outside the module. In AGMD, a stagnant air gap is introduced for condensation on a cold surface, while SGMD uses a cold inert gas to sweep vapors to con-dense externally [16]. DCMD is chosen for this hy-brid system due to its simplicity, high water flux, and compatibility with low-grade solar thermal energy, making it suitable for integration with the FO stage, as illustrated in Figure 6. Although scaling and mem-brane wetting may occur at very high solute concen-trations, MD effectively regenerates the draw solution for reuse, maintaining a closed-loop operation.

### 3.3. Mathematical calculations

The calculations are divided into two steps; the first one is Forward Osmosis and Direct Contact Membrane Distillation.

#### 3.3.1. Forward Osmosis

a. Water Flux (Jw)

$$\mathrm{Jw} = \Delta \mathrm{V} / (\mathrm{A} \times \Delta \mathrm{t}) \quad [17] \qquad (1)$$

Where:

V = Volume of permeate (L)
A = Membrane area ($m^2$)
t = Time (h)

This relationship determines the amount of permeate passing through the membrane under the osmotic pressure difference between the feed and draw solutions.

b. Recovery Ratio (%)

$$\text{Recovery Ratio} = ((\text{V permeate})/(\text{V feed})) \times 100 \quad [18] \qquad (2)$$

This parameter is crucial for evaluating the desalination efficiency and the reduction of brine discharge. Increasing recovery indicates better utilization of the feed stream and higher overall system efficiency.

During FO operation, the concentration of salts in the feed increases as water permeates to the draw solution, while the draw solution becomes diluted. These concentration variations directly affect the osmotic driving force and, consequently, the flux and recovery rate.

#### 3.3.2. MD stage (Distillate & Thermal Energy)

Thermal Energy Requirement (Q)

a. Sensible heating

$$\mathrm{Q} = \mathrm{m} \times \mathrm{Cp} \times \Delta \mathrm{T} \quad [19] \qquad (3)$$

Where:

m = mass of water (kg)
Cp = specific heat of water (kJ/(kg·°C))
ΔT = temperature difference (°C)

In this step, the draw solution is heated from its initial temperature to the operating temperature of the MD unit.

With $\eta_{sens}$:

$$Q_{sens,net} = (1 - \eta_{sens}) \times Q_{sens} \quad [20] \qquad (4)$$

Where $\eta_{sens}$ represents the fraction of heat recovered through the heat exchanger.

b. Latent Heat of Vaporization

After heating, part of the draw solution undergoes phase change as water molecules evaporate and condense on the permeate side of the membrane, producing distillate water. The latent heat ($Q_{lat}$) represents the energy needed to evaporate a specific mass of water and is given by:

$$Q_{lat} = \mathrm{M} \times \mathrm{L} \quad [21] \qquad (5)$$

3.3.3. Specific Energy Consumption (SEC)

$$\mathrm{SEC} = \frac{Q_{MD,net}}{V_{dis}} \quad [22] \qquad (6)$$

Where:

$Q_{MD,net}$ = net heat/energy supplied to the MD process

$V_{dis}$ = produced distillate volume ($m^3$)

The Specific Energy Consumption (SEC) quantifies the energy required to generate one unit of distilled water. Lower SEC values correspond to higher system energy efficiency.

3.3.4. Gained Output Ratio (GOR)

$$\mathrm{GOR} = \frac{m_{distillate} \times H_{vaporization}}{Q_{input}} \quad [17] \qquad (7)$$

Where:

$m_{distillate}$ = Mass of distillate (kg)

$H_{vaporization}$ = Latent heat of vaporization (kJ/kg)

$Q_{input}$ = Thermal energy supplied (kJ)

The Gained Output Ratio (GOR) evaluates the thermal efficiency of the MD process by comparing the energy utilized for vapor generation with the total energy input. A higher GOR implies better energy recovery

## 4. Results and Discussion

4.1. Discussion and outcome

The proposed solar-powered hybrid FO–MD system demonstrates the potential to achieve significantly higher water recovery compared to conventional RO systems, which are typically limited to 40–50% recovery under similar conditions. Achieving recovery rates exceeding 80% indicates that the system can produce a substantially greater volume of freshwater from the same seawater brine feed, while considerably reducing the amount of concentrated brine requiring disposal. This reduction in brine volume mitigates environmental impacts on marine ecosystems, addressing one of the major sustainability challenges of conventional desalination.

In the FO stage, the brine salinity is effectively reduced, lowering the osmotic pressure and preparing the feed for subsequent treatment through MD. The MD stage then utilizes low-grade thermal energy from solar sources to produce high-purity water. By combining these two processes, the system achieves improved energy efficiency compared to standalone MD or conventional RO, as less energy is required to achieve similar or higher water recovery.

Overall, the integration of FO and MD offers both technical and operational advantages, including enhanced freshwater production, lower specific energy consumption, and reduced environmental impact. The use of renewable solar energy further reinforces system sustainability, making it suitable for decentralized or off-grid applications. These results highlight the system's potential as a practical solution for efficient brine management, water desalination, and resource recovery, in alignment with circular water strategies and zero liquid discharge (ZLD) goals.

4.2. Challenges and Recommendations

The proposed FO–MD hybrid system faces several challenges and limitations. Selecting suitable FO and MD membranes that can withstand high salinity while maintaining long-term performance is critical, and maintaining an effective and reusable draw solution in the FO stage without excessive energy input remains complex. The performance of the MD unit depends on consistent low-grade heat, which requires reliable solar integration or heat recovery systems. Ensuring stable operation and compatibility between FO and MD components demands careful control and balancing of flow rates and pressures. Moreover, translating the design into a scalable solution may face constraints related to cost, infrastructure, and long-term membrane performance.

To address these challenges, it is recommended to implement the system on a pilot scale to evaluate real-world performance, membrane fouling behavior, and operational stability. Integrating energy recovery techniques, such as low-grade waste heat or solar thermal collectors, can enhance energy efficiency and reduce operational costs. Advanced real-time monitoring of salinity, temperature, and flow rate should be implemented to optimize control and maintain consistent water recovery. Further research on membrane materials can improve the performance and durability of both FO and MD membranes under high-salinity brine conditions. Additionally, after the FO stage, the system could be optimized to recover valuable minerals from the brine, contributing to resource recovery and supporting circular water management strategies, making the system technically feasible, sustainable, and operationally efficient.

## 5. Conclusion

The solar-powered FO–MD hybrid system presents a sustainable and efficient ap-proach for brine manage-ment, water desalination, and resource recovery. By inte-grating forward osmosis and membrane distillation, the system can achieve high water recovery, reduce brine salinity, minimize energy consumption, and lower environmental impacts compared to conventional methods. Its modularity, opera-tional efficiency, and compatibility with renewable energy make it a promising so-lution for future desalination projects, particularly in off-grid or arid regions. This study demonstrates the technical feasibility and practical potential of the hybrid FO–MD system as a forward-looking strategy aligned with circular water man-agement and zero liquid discharge goals.

## 7. Conflicts of Interest

The authors declare no conflicts of interest.

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